EXOTIC ELLIPTIC SURFACES WITHOUT 1-HANDLES

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Abstract. In this article, we consider a sufficient condition that a knot-surgery or log-transformation of $E(n)$ admits a handle decomposition without 1-handles. We show that if K is a knot that the bridge number is $b(K) \leq 9n$, then the knot-surgery $E(n)_K$ of the elliptic surface $E(n)$ admits a handle decomposition without 1-handles. This means that if $\gcd(p,q) = 1$, and $\min\{p,q\} \leq 9$, then $E(1)_{p,q}$ admits a handle decomposition without 1-handles. We also show that if $\gcd(p,q) = 1$, $\min\{p,q\} \leq 4$, then the double log-transformation $E(n)_{p,q}$ admits a handle decomposition without 1-handles for any positive integer n.

1. Introduction

1.1. **Handle decomposition of a 4-manifold.** A long-standing interesting problem of smooth 4-manifolds is whether any simply-connected closed smooth 4-manifold admits a handle decomposition without 1-handles or 3-handles.

The following is the classical main problem about 1-handles and 3-handles of closed smooth 4-manifolds.

Problem 1.1 (Kirby's problem [7]). Let X be a simply-connected closed smooth 4-manifold.

- Does X admit a handle decomposition without 1-handles?
- Does X admit a handle decomposition without 1-handles and 3-handles?

Several affirmative evidences about Problem 1.1 as below has been known. These problems are essentially difficult to resolve. In fact, if any homotopy S^4 admits a handle decomposition without 1-handles and 3-handles, the 4-dimensional smooth Poincaré conjecture is resolved affirmatively.

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There are some simply-connected closed smooth 4-manifolds which have so complicated handle decompositions, and it does not seem that all simply-connected closed 4-manifolds admit a handle decomposition without 1-handles and 3-handles. Let $E(n)$ be the elliptic surface with $12n$ Lefschetz singularities. For instance, Harer, Kas and Kirby in [6] conjectured that the Dolgachev surface $E(1)_{2,3}$ requires at least a 1-handle. Gompf also conjectured in [5] that some $E(n)_{p,q}$ might require at least one 1-handle. In this article, we consider a sufficient condition that a 4-manifold $E(n)_K$ or $E(n)_{p,q}$, which it is known some of these manifolds are exotic to $E(n)$, has a handle decomposition without 1-handles. Whether we can eliminate 3-handles after eliminating 1-handles is a more subtle problem, we do not deal with it in this article.

1.2. **Handle decomposition of $E(n)_K$ or $E(n)_{p,q}$.** Here, we write a history on sufficient conditions of $E(n)_K$, $E(n)_{p,q}$ or its blow-ups that admit handle decompositions without 1-handles or 3-handles. Harer-Kas-Kirby's conjecture in [6] was negatively resolved by Yasui in [13] and Akbulut in [1] independently. Yasui in [13], constructed handle decompositions of $E(n)_{p,q}$ for $(p,q) = (2,3), (2,5), (3,4)$ and $(4,5)$ for any positive integer n without 1-handles. Akbulut also in [2] proved that a knot surgery $E(1)_{K_n}$ of $E(1)$ for each of an infinite family $\{K_n\}$ of knots admits a handle decomposition without 1-handles and 3-handles. It gives an exotic family of $E(1)$. Sakamoto constructed a handle decomposition of $E(1)_{2,7}$ without 1-handles in [10]. Recently, Monden and Yabuguchi in [9] announced that $E(1)_{2,q}$ admits a handle decomposition without 1-handles and 3-handles. Kusuda in [8] gave some handle decompositions $E(n)_{5,6}$, $E(n)_{6,7}$, $E(n)_{7,8}$ and $E(n)_{8,9}$ of the double log-transformation without 1-handles for $n \geq 4$, $n \geq 5$, $n \geq 9$, and $n \geq 24$ respectively.

1.3. **Main results.** In this article, we prove a wider condition for a knot-surgery of $E(n)$ to admit a handle decomposition without 1-handles. Here $b(K)$ is the bridge number of a knot K. The following is the main theorem of this article.

Theorem 1.2. Let K be a knot in S^3 with $b(K) \leq 9n$. Then $E(n)_K$ admits a handle decomposition without 1-handles.

This theorem is applicable to the case of double log-transformation of $E(1)$, because of $E(1)_{T_{p,q}} = E(1)_{p,q}$. See [4] for this equality. Then we obtain the following immediately. Here, for relatively prime integers p, q, $T_{p,q}$ stands for the right-handed torus knot for $pq > 0$. If $pq < 0$, $T_{p,q}$ represents the left-handed torus knot.

Corollary 1.3. Let p, q be relatively prime positive integers. Then if $\min\{p, q\} \leq 9$, then $E(1)_{p,q}$ admits a handle decomposition without 1-handles.

Proof. The bridge number of a torus knot $T_{p,q}$ is $\min\{p, q\}$. Thus, equality $E(1)_{T_{p,q}} = E(1)_{p,q}$ and Theorem 1.2 make $E(1)_{p,q}$ admit a handle decomposition without 1-handles. □

This result can also be partially extended to more general $E(n)_{p,q}$.

Theorem 1.4. Let n be a positive integer. Let p, q be relatively prime integers. If $\min\{p, q\} \leq 4$, then $E(n)_{p,q}$ admits a handle decomposition without 1-handles.

Recall that the case of $n = 1$ is already proven in Corollary 1.3. This theorem also encompasses the results of Yasui in [13]. To prove this proposition, we use the fact that any log-transformation is regarded as a "twisted" knot-surgery.

Here, we ask the following question instead of Harer-Kas-Kirby's conjecture.

Question 1.5. Does $E(1)_{10,11}$ admit a handle decomposition without 1-handles and 3-handles?

Acknowledgements

The first main result was essentially inspired by works of Akbulut and Yasui fifteen years ago. However, apparently it has been less known ever for experts. This article was inspired by talks of Yabuguchi and Kusuda in the workshop "Four Dimensional Topology" on October in 2024 held at Osaka. The author is grateful for their results so much. Also, he thanks Kouichi Yasui for giving some helpful advice for an early draft.

2. Preliminaries

2.1. **Knot-surgery.** Let X be a 4-manifold in which the trivial tubular neighborhood of a torus T is contained. Hence, we can take a diffeomorphism $\nu(T) \cong T \times D^2$. Here, $\nu(A)$ is a tubular neighborhood of a submanifold A. Let $K \subset S^3$ be a knot. Then, *knot-surgery* is defined by the following surgery:

$$X_K := (X - \nu(T)) \cup_\phi (S^3 - \nu(K)) \times S^1.$$

The gluing map $\phi : \partial\nu(K) \times S^1 \to T \times \partial D^2$ satisfies $\phi(m) = \lambda_1$, $\phi(s) = \lambda_2$, $\phi(l) = d$, where $m, l \subset \partial\nu(K) \times \{\mathrm{pt}\}$ are the meridian and longitude of K respectively, s is $\{\mathrm{pt}\} \times S^1$, d is the meridian circle $\{\mathrm{pt}\} \times \partial D^2$ of T, and $\lambda_1, \lambda_2 \subset T \times \{\mathrm{pt}\}$ are two simple closed circles generating $H_1(T)$.

2.2. **Bridge presentation of knot.** Let K be a knot in S^3. If there exists a decomposition $S^3 = B_0^3 \cup B_1^3$ with $B_0^3 \cap B_1^3 = \partial B_0^3 = \partial B_1^3$, such that B_i^3 is the 3-ball and the intersection $K \cap B_i = K_i$ is a set of n boundary-parallel arcs in the 3-ball. This is called an *n-bridge presentation* of K. We define the bridge number

$$b(K) = \min\{n | K \text{ admits an } n\text{-bridge presentation}\}.$$

A knot with $b(K) = 1$ is the trivial knot. For example, a knot with $b(K) = 2$ is called a 2-bridge knot.

It is easy to show that an n-bridge presentation of K has a normal form as in Figure 1.

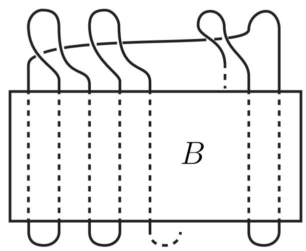

FIGURE 1. A normal form of n-bridge knot. B is a pure braid.

Fact 2.1. Let K be a knot with an n-bridge presentation. Then, we can find a bridge presentation as Figure 1 using a pure braid B of a $2n$-string.

The set of generators of the pure braid group PB_n with an n-string is well-known.

Fact 2.2. The pure braid group PB_n has the following set of generators $\{\mathcal{T}_{i,j} \mid 1 \leq i < j \leq n\}$ as in Figure 2.

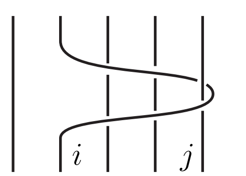

FIGURE 2. A generator element $\mathcal{T}_{i,j}$ of PB_n.

2.3. **Handle decomposition of knot-surgery.** In this section, we construct a handle decomposition of knot-surgery. This is due to [3, 11, 12]. Let X be a 4-manifold which $T^2 \times D^2$ is contained. We may assume that we regard the handle decomposition of X as a decomposition constructed by attaching several handles over $T^2 \times D^2 = h^0 \cup h_1^1 \cup h_2^1 \cup h^2$.

Here, h^k stands for a k-handle. The handle decomposition of $T^2 \times D^2$ is as in Figure 3.

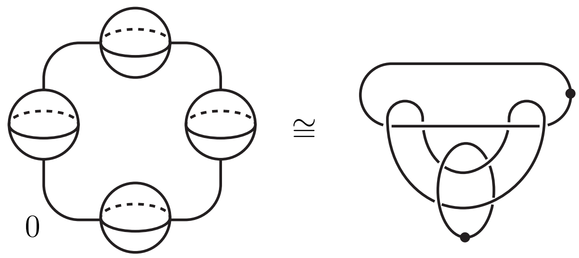

FIGURE 3. A handle decomposition of $T^2 \times D^2$.

First, we insert 2-/3-canceling pairs and 1-/2-canceling pairs in the positions in the second and third pictures in Figure 4. We deform the diagram as in Figure 4, which presents a deformation in the case of two pairs of 2-/3-canceling handles. In general, similarly, we can also deform the diagram in the case where the number of 2-/3-canceling pairs is $n-1$.

Second, we use an n-bridge presentation of K in a normal form using a $2n$-pure braid B as shown in Figure 1. We remove $T^2 \times D^2$ from X. According to the presentation $B = \mathcal{T}_{i_1,j_1}\mathcal{T}_{i_2,j_2}\cdots,\mathcal{T}_{i_r,j_r}$, we deform the surgery diagram of the boundary. For example, $\mathcal{T}_{1,2}$ and $\mathcal{T}_{3,4}$ are isotopies by twisting. As another example, the deformation in the case of $\mathcal{T}_{2,5}$ is shown in Figure 5. These are the boundary diffeomorphisms over $\partial(S^3 - \nu(K)) \times S^1$. Gluing $(S^3 - \nu(K)) \times S^1$ to the boundary, we obtain the diagram of X_K. In the last diagram in Figure 4, we call a 0-framed 2-handle located in the center of the diagram a *centered 0-framed 2-handle* in the diagram of $(S^3 - \nu(K)) \times S^1$.

2.4. **Log-transformation.** Here, we give a short review of the log-transformation. Let $T^2 \subset X$ be an embedded torus with the trivial normal bundle. The boundary of the tubular neighborhood $\nu(T^2) \cong T^2 \times D^2$ of T^2 is a 3-torus T^3. The generator set of $H_1(\partial(T^2 \times D^2))$ is represented by $\{d, \lambda_1, \lambda_2\}$, where $d = \{\mathrm{pt}\} \times \partial D^2$, and $\{\lambda_1, \lambda_2\}$ presents a generator set of $H_1(T^2 \times \{\mathrm{pt}\})$. Then we define a map $\phi : T^2 \times \partial D^2 \to T^2 \times \partial D^2$ as follows:

$$\phi(d) = p' \cdot \gamma + p \cdot d,$$

where p and p' are relatively prime and γ is some primitive element in H_1 of the fiber torus with $\gamma = b \cdot \lambda_1 + c \cdot \lambda_2$ for some integers b, c. In this paper, for two simple closed (homologically non-trivial) curves x_1, x_2 in a 2-torus and for $a_1, a_2 \in \mathbb{Z}$, the notation $a_1 \cdot x_1 + a_2 \cdot x_2$ stands for an isotopy class of a simple closed curve representing the element

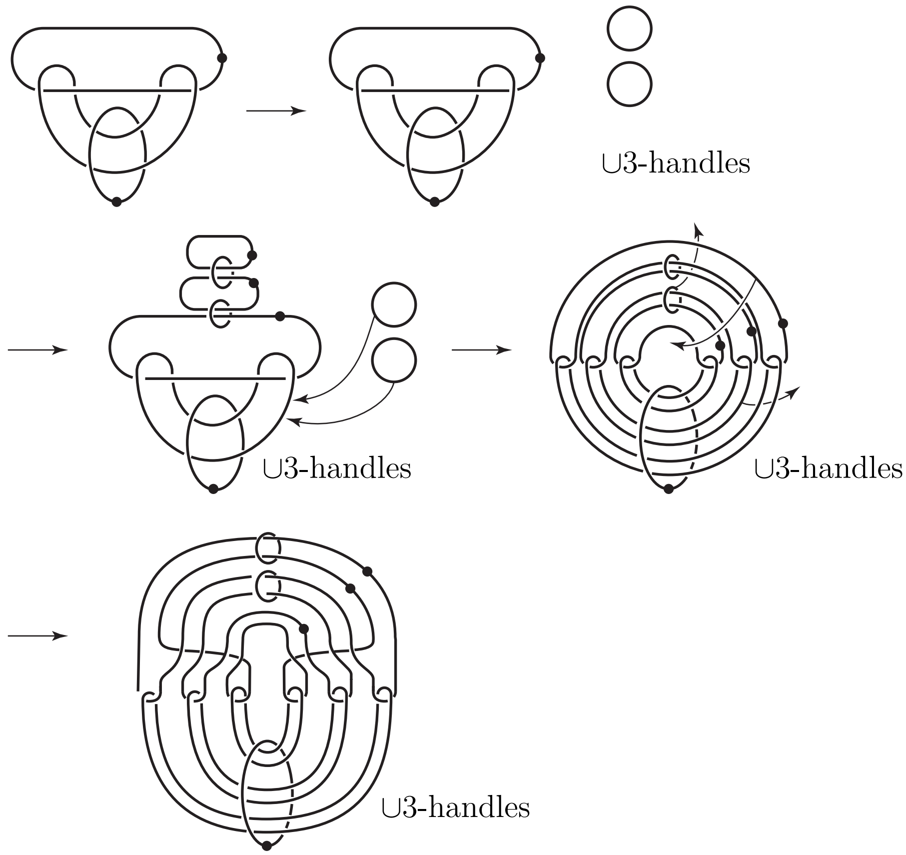

FIGURE 4. A deformation of the handle diagram of $T^2 \times D^2$. All the components with no dots are 0-framed 2-handles.

$a_1[x_1] + a_2[x_2]$ in H_1 of the torus. If the circle is one component, then $\gcd(a_1, a_2)$ must be 1.

Then the *log-transformation* along T^2 is the following surgery:

$$X_{\gamma,p',p} := (X - \nu(T^2)) \cup_\psi T^2 \times D^2.$$

Here p is called a *multiplicity* of log-transformation.

We assume $\pi_1(X - \nu(T^2)) = e$. The diffeomorphism class of the log-transformation depends only on the multiplicity, because of this π_1-condition. We denote the log-transformation by X_p. When we perform the log-transformation along two parallel tori with multiplicities p, q respectively, we say the surgery *double log-transformation*. We then denote the result as $X_{p,q}$.

Here we state a relation between the double log-transformation with multiplicities p, q and the knot-surgery of $T_{p,q}$. Let $T \subset X$ be an

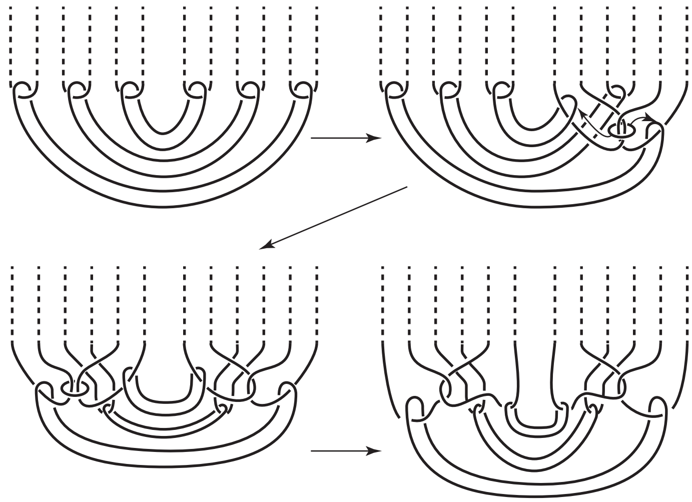

FIGURE 5. A deformation of diagram for a generator element $\mathcal{T}_{2,5}$ in PB_6

embedded torus with the trivial normal bundle. Let $d, \lambda_1, \lambda_2, m, l, s$ be the same as the things defined in Section 2.1.

Proposition 2.3. The log-transformation $X_{p,q}$ is diffeomorphic to $(X-\nu(T))\cup_{\phi'}(S^3-\nu(T_{p,q}))\times S^1$. The gluing map $\phi' : \partial(S^3-\nu(T_{p,q}))\times S^1 \to \partial\nu(T)$ satisfies $\phi'(m) = d$, $\phi'(l) = \lambda_1 - pq\cdot d$, and $\phi'(s) = \lambda_2$.

Proof. The manifold $S^3 - \nu(T_{p,q})$ is diffeomorphic to the twice Dehn surgery of $S^1 \times D^2$ along two parallel curves f_1, f_2 parallel to $\lambda_1 = S^1 \times \{\text{pt}\}$ as shown in Figure 6. The slopes of the Dehn surgeries are p/u and q/v where p, q are degrees about the meridians of f_1, f_2 and satisfy $pv + qs = 1$. A slope r/s of $T_{p,q}$ corresponds to a slope $r/s - pq$ of the dotted circle in the Seifert structure in the middle picture of Figure 6. By taking the product of S^1, $(S^3-\nu(T_{p,q}))\times S^1$ is diffeomorphic to $(T^2\times D^2)_{(\lambda_1,u,p),(\lambda_1,v,q)}$. Here, the S^1-direction is λ_2.

The meridian circle of $T_{p,q}$, i.e., ∞-slope in $\partial\nu(T_{p,q})$ is mapped to the ∞-slope of the dotted circle in Figure 6. The longitude circle (0-slope) of $T_{p,q}$ is mapped to a circle representing $\lambda_1 - pq\cdot d$ in $\partial\nu(T)$. The S^1-direction in $(S^3-\nu(T_{p,q}))\times S^1$ is mapped to λ_2.

Hence, the gluing map $\phi' : \partial(S^3-\nu(T_{p,q}))\times S^1 \to \partial\nu(T)$ satisfies $\phi'(m) = d$, $\phi'(l) = \lambda_1 - pq\cdot d$, and $\phi'(s) = \lambda_2$. Then we have the following.

$$\begin{aligned} X_{p,q} &= (X-\nu(T))\cup(D^2\times T^2)_{p,q} \\ &\cong (X-\nu(T))\cup_{\phi'}(S^3-\nu(T_{p,q}))\times S^1. \end{aligned}$$

□

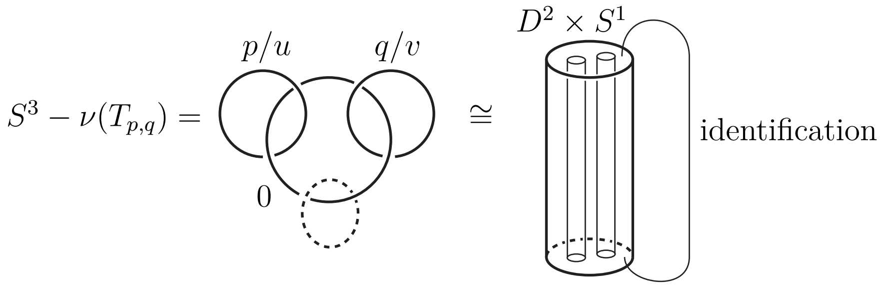

FIGURE 6. A diffeomorphism from $S^3 - \nu(T_{p,q})$ to a double log-transformation along two curves.

3. Proofs

3.1. **The global monodromy of $E(n)$.** Let $E(n)$ be the elliptic surface with $12n$ Lefschetz singularities. We divide $E(n)$ a neighborhood of general fiber T and the complements as in $(E(n) - \nu(T)) \cup T \times D^2$. We perform the knot-surgery in $E(n) - \nu(T)$

$$E(n)_K \supset (E(n) - \nu(T))_K$$

Let two elements a, b be $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, and $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \in SL(2, \mathbb{Z})$. The global monodromy of $E(n)$ is $(ab)^{6n}$ and can be deformed as follows

$$\begin{aligned} (ab)^{6n} &= (abababababab)^n = (a(aba)a(aba)a(aba))^n \\ &= (a^2ba^3ba^3ba)^n \sim (a^3ba^3ba^3b)^n. \end{aligned} \tag{1}$$

3.2. **Proof of Theorem 1.2.** Let K be a knot with $b(K) \leq 9n$. Since $E(n)$ has $12n$ Lefschetz singularities, the handle decomposition of $E(n)$ consists of the union of $T \times D^2 \cong h^0 \cup h_1^1 \cup h_2^1 \cup h^2$, $12n$ (-1)-framed 2-handles and $h^2 \cup h_1^3 \cup h_2^3 \cup h^4$. We obtain a handle decomposition around a general fiber T as in Figure 7 (a). For simplicity, here it is the case of $n = 1$. From the computation of monodromy (1), the parallel $9n$ -1-framed 2-handles in the figure correspond to the vanishing cycles for the element a. Another -1-framed 2-handle corresponds to the vanishing cycle for b. By removing $T \times D^2$ and gluing $(S^3 - \nu(K)) \times S^1$ as shown in Figure 7 (b), the meridian circle d of $T \times D^2$, through the boundary diffeomorphism, is mapped to the meridian of a 1-handle of $(S^3 - \nu(K)) \times S^1$ as illustrated in Figure 7. Here, sliding some -1-framed 2-handles over 0-framed 2-handles we can move them to each of the meridians of other 1-handles. We obtain the handle diagram in Figure 7 (c). Since we can give a handle decomposition having $b(K) + 1$ 1-handles in $(S^3 - \nu(K)) \times S^1$, all 1-handles in the picture can be canceled from the condition $b(K) \leq 9n$. Since $E(n) - \nu(T)$ has no

1-handles, $E(n)_K$ admits a handle decomposition without 1-handles.□

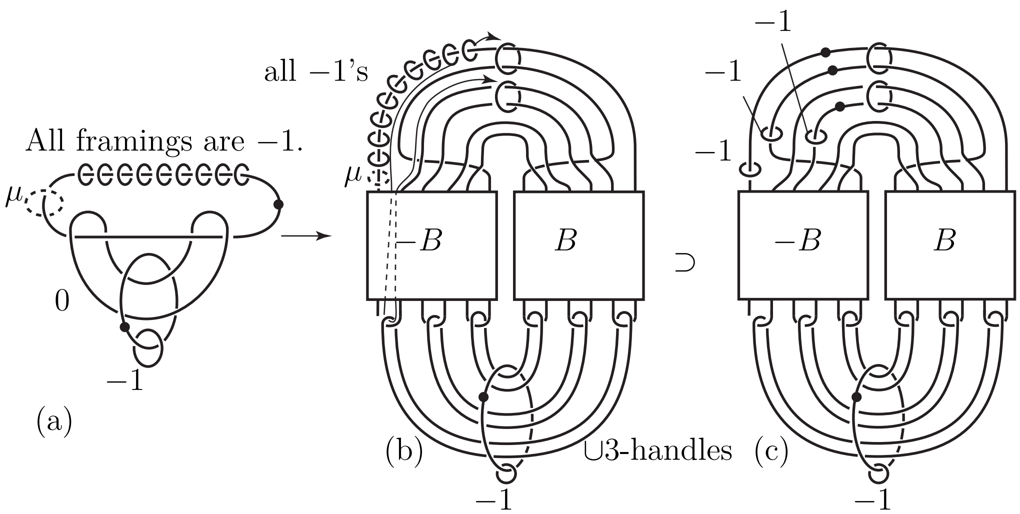

FIGURE 7. A knot-surgery near a regular fiber with 9+1 vanishing cycles. Here B is a pure braid.

Since $b(T_{p,q}) = \min\{p, q\}$, the condition $\min\{p, q\} \leq 9n$ gives a handle decomposition of $E(n)_{T_{p,q}}$ without 1-handles.

3.3. **Proof of Theorem 1.4.** Let p, q be relatively prime positive integers. We decompose $E(n)_{p,q} = ((S^3 - \nu(T_{p,q})) \times S^1) \cup_{\phi'} (E(n) - \nu(T))$ according to Proposition 2.3. The 1-handles of $E(n)_{p,q}$ are induced by the 1-handles of $(S^3 - \nu(T_{p,q})) \times S^1$. Over the meridian of the 1-handle of the S^1-direction, a -1-framed 2-handle is attached from $E(n) - \nu(T)$ through ϕ'. Hence, this 1-handle is canceled. Over the 1-handle for the meridian m in $(S^3 - \nu(T_{p,q})) \times S^1$, a section $E(n) - \nu(T)$ is attached. The section gives a $-n$-framed 2-handle h_0 in $E(n) - \nu(T)$.

We create a 0-framed 2-handle h_1 as a 2-/3-canceling pair in the diagram. Slide the 0-framed 2-handle h_0 over the $-n$-framed 2-handle as shown in Figure 8. Then we obtain two meridians h_0, h_1 with linking $-n$.

Here, we use the parallel $9n$ 2-handles attached over λ_1. According to the process as illustrated in Figure 9, we can construct a chain type link with all framings -2 and with the length $9n - 1$. We call this link a -2-*chain with length* $9n - 1$. Removing the $(n + 2)$-nd component in the -2-chain, we get a -2-chain with length $n + 1$.

Appropriately sliding the 2-component 2-handles (h_0, h_1) with linking $-n$ over the -2-chain with length $n+1$, we can untie the $-n$-linking as shown in Figure 10. Then, the framings of the two 2-handles (h'_0, h'_1)

become

$$\begin{cases}(-2n-1,-2n-1) & n \text{ odd}\\ (-2n,-2n-2) & n \text{ even.}\end{cases}$$

After this, removing 2-handles for the -2-chain with length $n+1$ from the diagram, we obtain two meridian 2-handles. We slide one of the 2-handles over the centered 0-framed 2-handle to realize the canceling position of two 1-handles. Recall the definition of the centered 0-framed 2-handle in Section 2.3. Therefore, we can cancel all 1-handles of $E(n)_{2,q}$.

Next, we create a 0-framed 2-handle h_2 as a 2-/3-canceling pair in the diagram. Slide the 0-framed 2-handle over h_0' in the similar manner. Then we obtain two meridian 2-handles with linking $-2n-1$, or $-2n$. Let $-N$ denote $-2n-1$, or $-2n$. We untie the linking by using the rest of -2-chain with length $8n-3$. The we obtain a meridian 2-handle h_2'. Here we remove the $(N+2)$-nd component in the rest of -2-chain, then we use a -2-chain with length $N+1$. Then the framings of the two 2-handles become

$$\begin{cases}(-2N-1,-2N-3) & n\text{: odd}\\ (-2N,-2N-2) & n\text{: even.}\end{cases}$$

Then we obtain three meridian 2-handles (h_0', h_1', h_2'). The framings are

$$\begin{cases}(-4n-3,-2n-1,-4n-3) & n\text{: odd}\\ (-4n,-2n-2,-4n-2) & n\text{: even.}\end{cases}$$

Using the centered 0-framed 2-handles, we distribute the three components to each meridian of 1-handles. These are canceling pairs. Therefore, we can cancel all 1-handles of $E(n)_{3,q}$.

Furthermore for the 2-handle h_1', we carry out the same process. That is, create a 0-framed 2-handle h_3, slide h_3 over h_1', we untie the linking by using a -2-chain. Then we obtain two meridians (h_1'', h_3'). We reduced $5n+8$-component from the $9n-1$-component -2-chain. Here $5n+8 \leq 9n-1$ holds for $n>2$. Even in the case of $n=2$, the last removing -2-framing is not needed, that is,

$$9n-1 = 17 = 3+1+5+1+7$$

holds. Then, we realize four meridian 2-handles of the 1-handles. Using the centered 0-framed 2-handles, we distribute the four components to each meridian of 1-handles. These are canceling pairs. Therefore, we can cancel all 1-handles of $E(n)_{4,q}$.

□

In the next section we prove the last theorem (Theorem **??**).

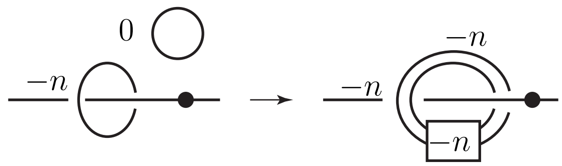

FIGURE 8. A handle slide. The box including $-n$ stands for the $-n$-full twist.

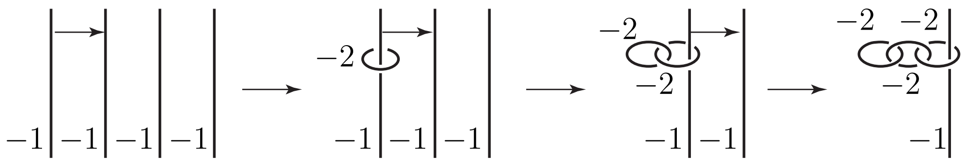

FIGURE 9. Handle slides to construct a chain of framed link.

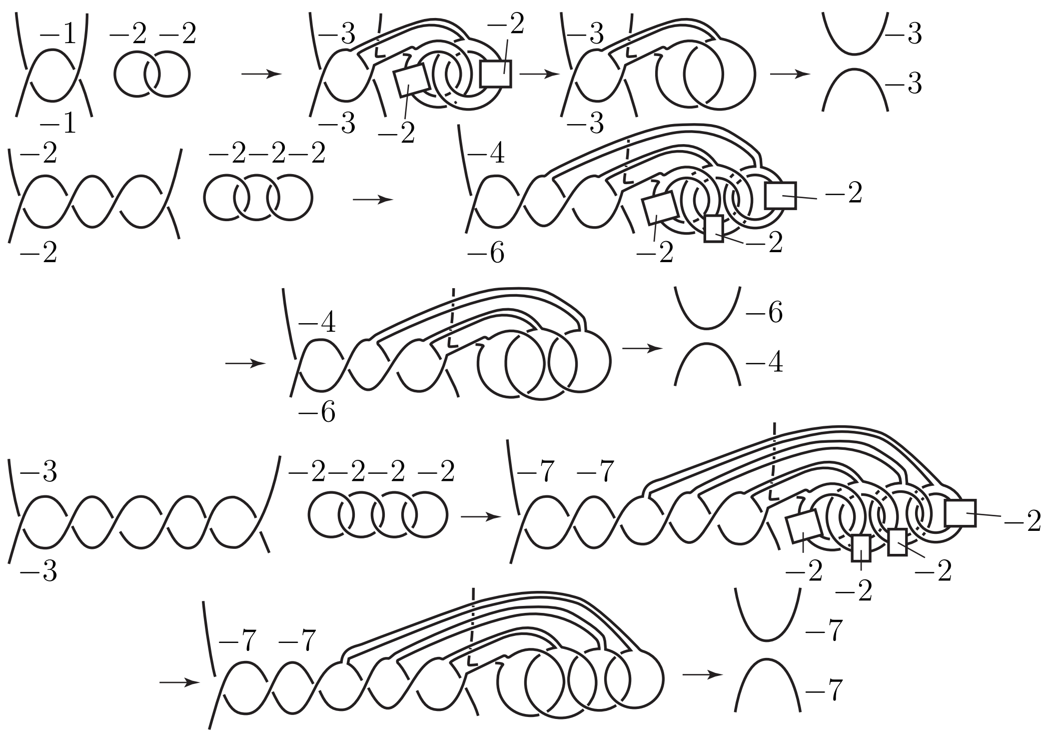

FIGURE 10. Unlinking processes of $n = 1$, 2, 3 using a -2-chain with length $n + 1$.

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