Divide knots & Kirby Melvin's grapes

§1 Divide knots (A'Campo)

例1 $T(p,q)$. $p,q>0$.

$z^q - w^p = 0$. singularity at $(0,0)$

$r(e^{ip\theta}, e^{iq\theta})$

実部

$r(\cos p\theta, \sin q\theta)$

Twisted torus knot [Dean]

$T(p,q;r,s)$. $p>q>0$, $p>r>0$

$T(4,3,2,1)$

obtained from $T(p,q)$

s-full twists along

r-parallel of strings

in the p-strings

Def. divide links (construction)

P : a generic properly immersed curve in the unit disk D^2

$\Downarrow$

$L(P)$ a link in S^3

(generic
No self tangency
No triple points)

D^2

$$\Rightarrow S^3 = \{ (x,v) \in D^2 \times T_x D^2 \mid |x|^2 + |v|^2 = 1 \}$$

$$\cup$$

$$L(P) = \{ (x,v) \in P \times T_x P \mid \quad " \quad \}$$

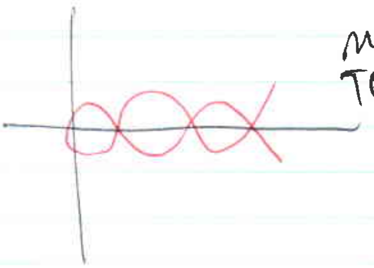
Idea 特異点の実モース化

「Real part of a good "perturbation" of the cpx curve singularity」

ex. $y^2 = x^{2n+1}$ の場合 $T(2, 2n+1)$

$y^2 = x(x-\varepsilon)^2(x-2\varepsilon)^2 \cdots (x-n\varepsilon)^2 = 0$

Lissajous 曲線 $\ell(2,7)$



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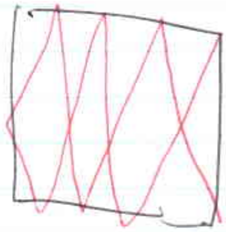
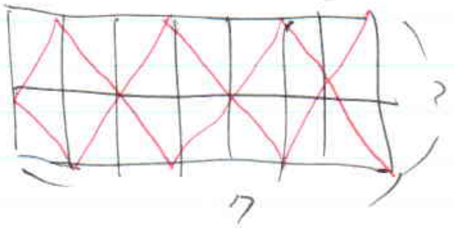
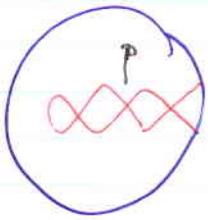


Fact (今西・平澤・小川) $\ell(p,q)$ presents $T(p,q)$

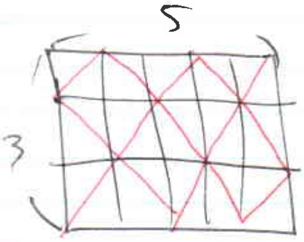
Billiard curve $B(2,7)$





$\Rightarrow L(P) = T(2,7)$

good=「必要な特異点をReal partに揃える」

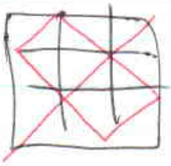


$\to T(3,5)$

$(p,q)=1$. $\#$交点 $= \frac{(p-1)(q-1)}{2}$

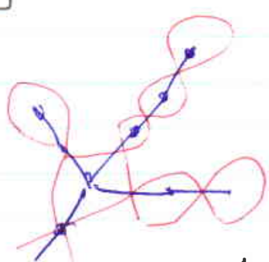
$\uparrow$

$u(K)$, $g_3(K)$, $g_4(K)$



$\to T(3,3)$ link も可.

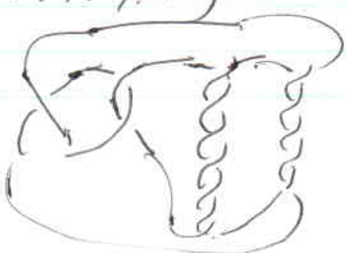
例



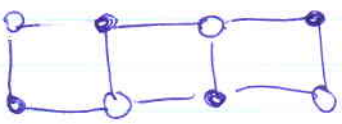
slalom

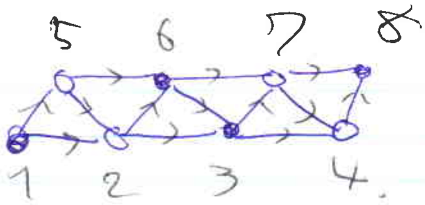
$(\#b_r - 2)$に

$pr(-1,-1,2,3,7,5)$



例

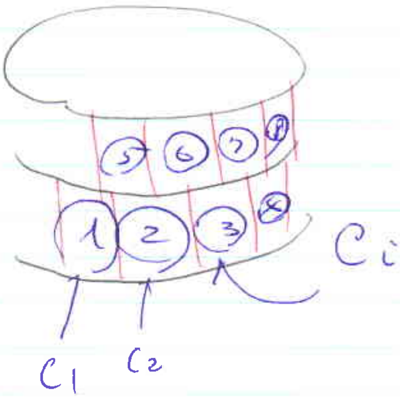




Numbering 下→上
左→右.

$\varphi = D_1 D_2 \cdots D_N$

D_i はさっきの ~~Number~~ surface の C_i に沿った に沿う right handed Dehn twist



φの homology への作用を行列で表示する。

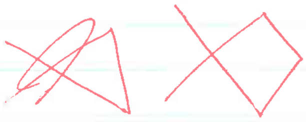
2g×2g 行列 $\quad B_i = I + \sum_j \sigma^i_j E_{ij}$

$N = 2g.$

$E_{ij} = \begin{array}{cc} & j \\ i & \begin{pmatrix} \cdot & \\ & 1 \end{pmatrix} \end{array}$ 他全て0

$$\sigma^i_j = \begin{cases} i \longrightarrow j & \Rightarrow +1 \\ i \longleftarrow j & \Rightarrow -1 \\ \text{o/w} & \Rightarrow 0 \end{cases}$$

trefoil



$1 \to 2$

$$B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$B_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

Basic Facts on divide knots (A'Campo, Rudolph, 平澤, 石川, 北村)

(1) $L(P)$ の成分数 $=$ #arc $+ 2$ #circle of P

(2) P is connected $\Rightarrow L(P)$ is fibered.

(x, v)
$\downarrow$
$(x, -v)$

(3) $L(P)$ is strongly invertible. $(\underset{\mathbb{Z}/2}{\circlearrowleft} S^3 - K)$

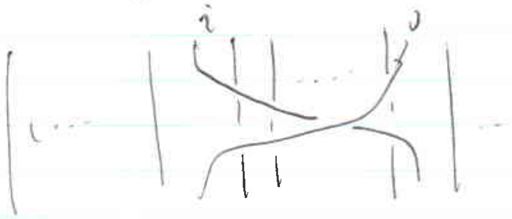
(4) $L(P)$ is a closure of strongly quasi positive braid

(5) $L(P)$ is a (quasi) knot $\Rightarrow u, g_3, g_4 =$ #交点数 of P

- quasi pos.

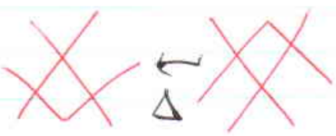
σ_i $w\sigma_i w^{-1}$ で生成される

- strongly quasi pos

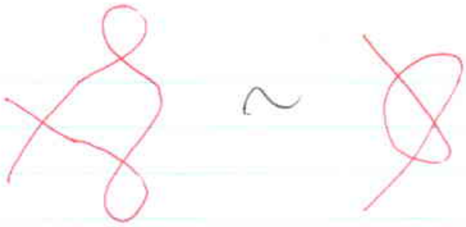


σ_{ij} の積でかけている braid

(6) $P_1 \underset{\Delta}{\sim} P_2 \Rightarrow L(P_1) = L(P_2)$



演習

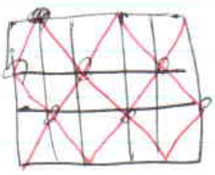


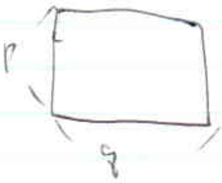
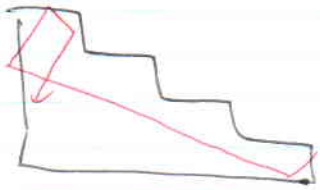
Pr(-2,3,3) T(3,4)

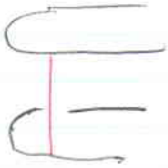
ex. Pr(-2,3,5) = T(3,5).

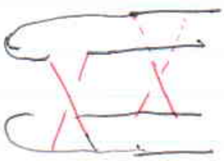
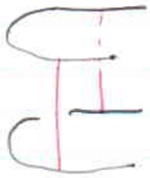
Today P is a Billiard curve of a rectangle or L-shaped.
gen

← この角はとらない

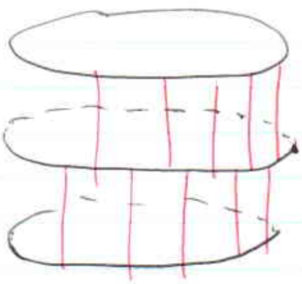


 or 

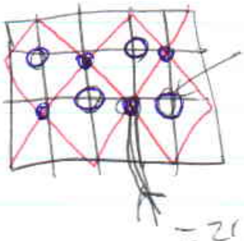
 の角をもつ場合 →  を加える 略 

 〃 →  〃 略 

よって T(3,5) は…

F(3,5) 

Monodromy φ of fiber surface



とくい点（複素の点で）ある。

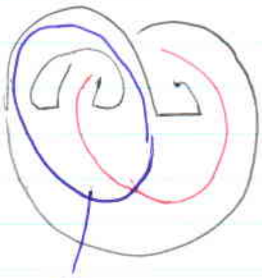
ここでは元々特異点ない

Dehn twist

Remark. $D_1 = U$, $D_2 = V$, $(UV)^6 = 1$.

$UVU = VUV \longleftrightarrow B_1B_2B_1 = B_2B_1B_2$

(組紐関係)



C1

$$B_1B_2 = \begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}1&0\\-1&1\end{pmatrix} = \begin{pmatrix}0&1\\-1&0\end{pmatrix}$$

$$\begin{pmatrix}0&1\\-1&1\end{pmatrix}^6 = 1 \qquad (\varphi_{pq})^{pq} = 1$$

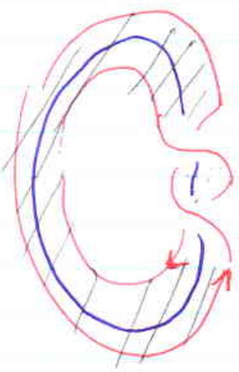
$$\det(I\cdot t - \varphi_*) = \Delta(t) \quad \text{Alex poly.}$$

$$\begin{vmatrix} t & -1 \\ 1 & t-1 \end{vmatrix} = t^2 - t + 1$$

$$= \Delta_{T(2,3)}(t)$$

§2. Fiber surface & Monodromy φ

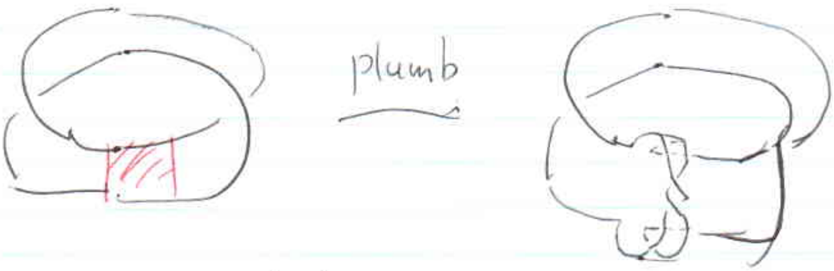
① +Hopf band. (= negatively twisted Hopf band)



$lk(\partial_+, \partial_-) = +1$

$\varphi = D_c$

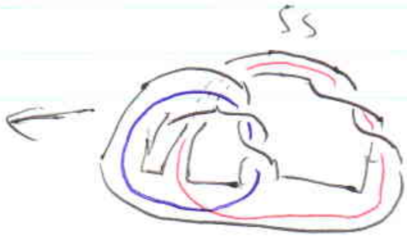
② Trefoil.



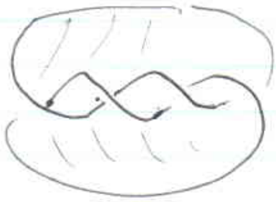
Map. torus of $\varphi = UV : T^2 \to T^2$

左手.

$g = 1$

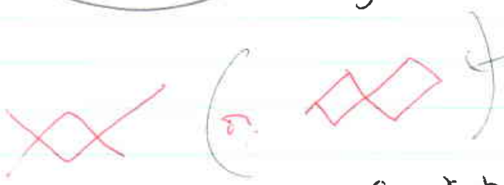


③ $T(4,2)$. ← Framed link



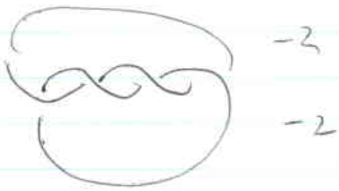
$2-4=2(1-g)-2$

$g=1$.



これからgenusは1がわからん

Fib surf の定める framing $=-2$.



→ Map. Torus of

$\varphi = UVU : T^2 \to T^2$

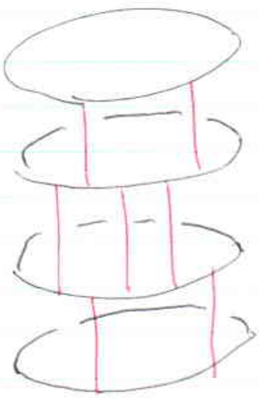
④. $T(3,3) = Pr(-1,2,2,2)$

$g=1$.

$= $ Map. Torus of

$\varphi = UVUV : T^2 \to T^2$

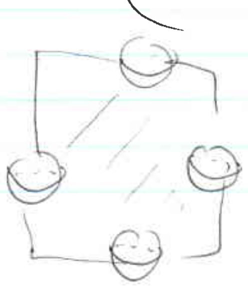
$\overset{\cup}{(UV)^2}$

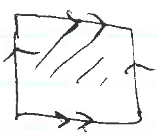


$g = \#$交点 $- \#$成分 $+ 1$.

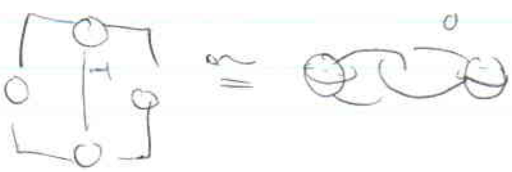
§3 genus 1 Lefschetz fib / D^2

$(T^2 \times D^2) \cup$ 2-h.dls.

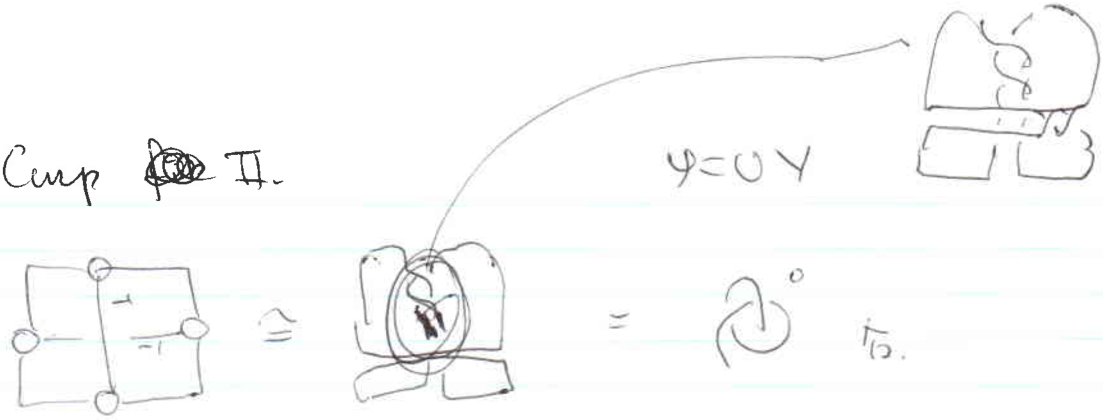




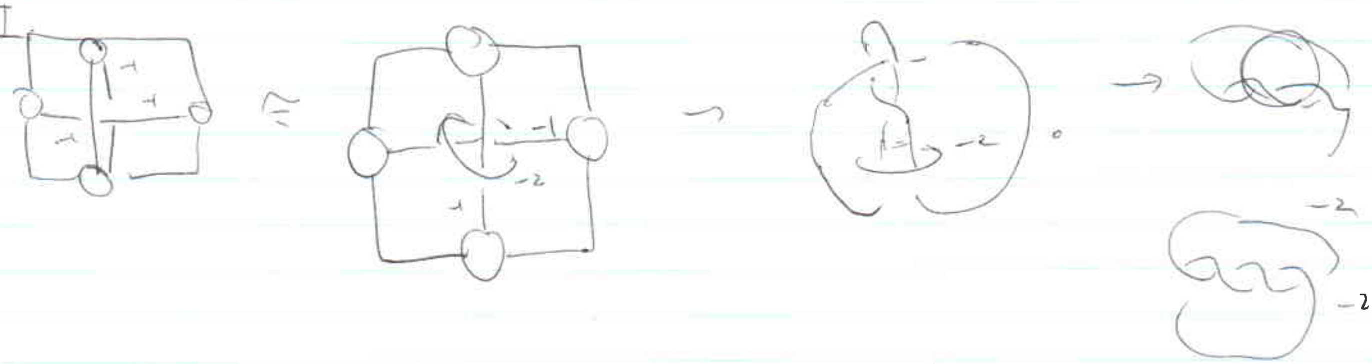
$\boxed{1}$ fishtail I $\varphi = U$.



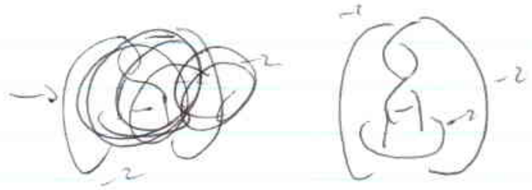
$\boxed{2}$ Cusp ~~[illegible]~~ II.



$\boxed{3}$ III



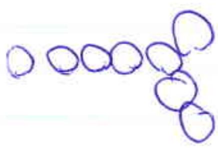
$\boxed{4}$ IV UVUV ≅ --

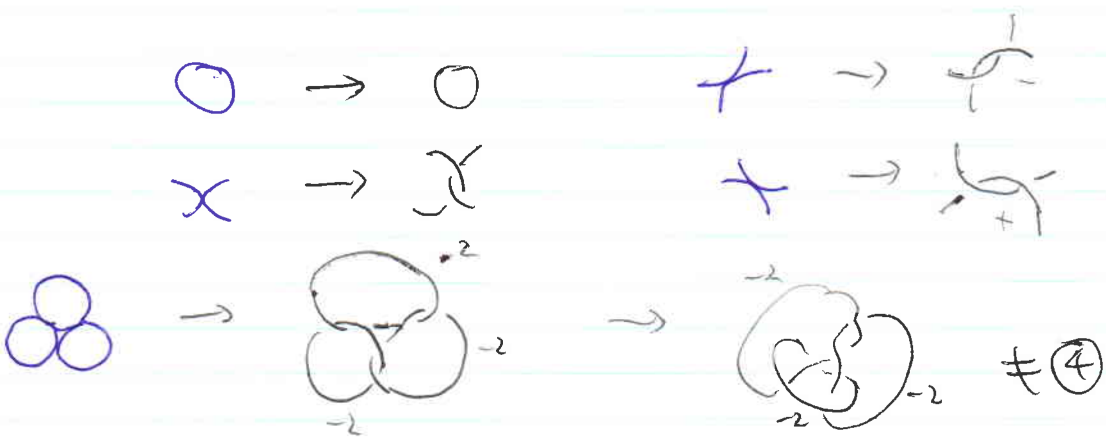


§3 Kirby-Melvin Grapes

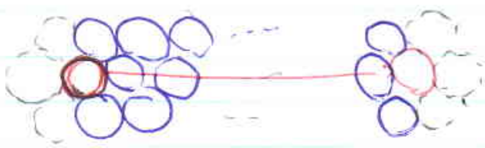
Def. A Grapes is a connected union of some circles the hexagonal circle packing of the plane.

ex.





Fact. Slide move



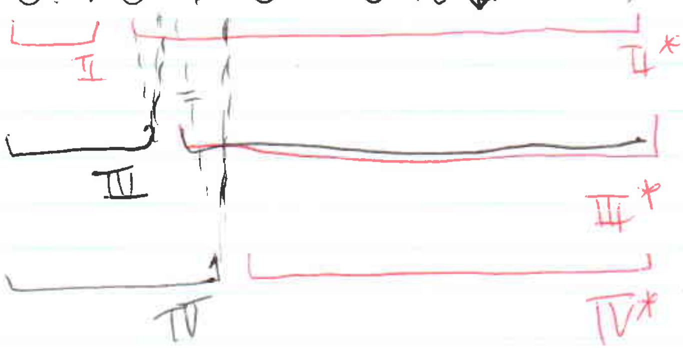
が空いている.

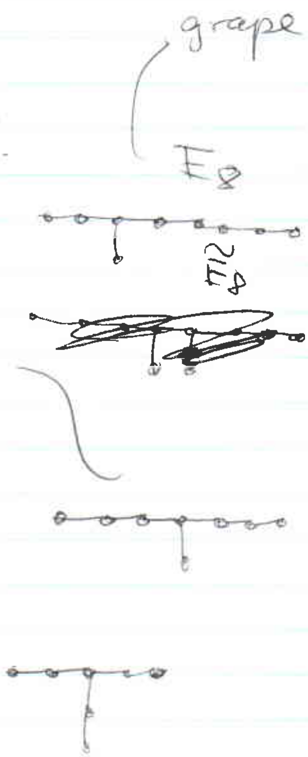
をにもっていける.

E_8 ‥ $(B^4, F(3,5))$ の 2 重 br cov.

Fig 1.3.

$E(1)$ $\varphi = U \cdot V\, U\, V\, U\, V\, U\, V\, U\, V\, U\, V = 1.$





$E(1) = \text{Cusp} \cup \mathrm{II}^*$

$\quad\quad\quad\quad\quad\quad\quad \cup$

$\quad\quad\quad\quad\quad\quad\quad E_8$