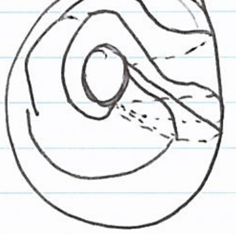
山田裕一

Torus knots. & lens space.

Torus Knots. geom. alg. $\mathbb{C}^2$. $z^q = w^p$

$S^3 \supset$



$T_{p,q}$

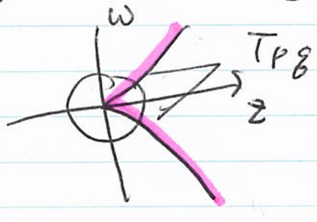
$\mathbb{C}^2 \supset S^3$

$(e^{pi\theta}, e^{qi\theta})$

$\theta \in \mathbb{R}/\underset{2\pi}{\mathbb{Z}}$

$\|$

$(\cos p\theta, \sin p\theta, \cos q\theta, \sin q\theta)$



Lissajous 曲線

実部 $(\cos p\theta, \sin q\theta)$

$\Sigma(4.95, 19)$.

Dehn surgery along K with slope γ $(= p[\mu] + q[\lambda])$.

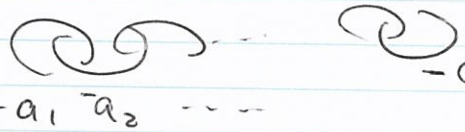
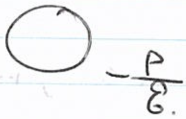
$M(K:\gamma) = E(K) \cup$ solid Torus.

$\gamma \longleftrightarrow$ meridian

 $= \Sigma(2,3,5)$ Poincare sphere

このとき、solid torus の core curve C を「この D-surgery」の ~~dual~~ dual knot」という

dual knot に沿う surgery で、$M(K.\gamma)$ から S^3 にもどすことができる。

$L(p,q) =$  $=$ 

$p/q = [a_1 \cdots a_n]^-$ $a_i > 1$,

$$= a_1 - \cfrac{1}{a_2 - \cfrac{\ddots}{- \frac{1}{a_n}}}$$

$$\frac{13}{9} = 2 - \frac{9}{5} = 2 - \frac{1}{\frac{5}{9}}$$

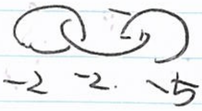
$$= 2 - \cfrac{1}{2 - \frac{1}{5}}$$

$L(p,q) \cong \pm L(p,q')$.

$\Leftrightarrow$ $q \equiv \pm q' \mod p$

$qq' \equiv \pm 1 \mod p$.

複号同順

$L(13,9) =$ 

$[5,2,2]^- = 5 - \cfrac{1}{2 - \frac{1}{2}}$

$L(13.9) \cong L(13.3)$

$9 \times 3 = 27 \equiv 1 \mod 13$

Thm [Moser '71]

$$M(T_{pq}; pq \pm 1) \cong -L(pq \pm 1, p^2)$$

$$(\cong -L(pq \pm 1, q^2))$$

Blow up. $\cancel{\mathbb{CP}^2}\,(\mathbb{C}^2, \overset{C}{0}) \rightsquigarrow (\mathbb{CP}^2 \# \overline{\mathbb{CP}^2}, \tilde{C})$

$$\mathbb{C}^2 \times \mathbb{CP}^1$$
$$\cup$$
$$U = \{ (z, w), [s:t] \mid zt = sw \}$$

$$\pi \downarrow \qquad \downarrow$$
$$(z, w)$$
$$\mathbb{C}^2 \supset z^q = w^p : C$$

$$\pi^{-1}(z, w) = \begin{cases} (z, w), [z:w] & \text{if } (z, w) \neq (0, 0) \\ \{[s:t] \mid \forall s, t\} & (z, w) = (0, 0) \end{cases}$$
$$\cong \mathbb{CP}^1$$

$$\mathbb{C}^2 \cong U_1 \hookrightarrow U \hookleftarrow U_2 \cong \mathbb{C}^2$$
$$(z, t) \longmapsto (z, zt)\,[1:t]$$
$$(ws, w)\,[s:1] \longleftarrow (w, s)$$
$$(z, t) \longmapsto (zt, \tfrac{1}{t})$$

$\cancel{\mathbb{CP}^2}$ $U = U_1 \cup U_2 / \sim$

$= \mathbb{CP}^1$ 上の $\mathbb{C}$束.

$= \overline{\mathbb{CP}^2} - pt$.

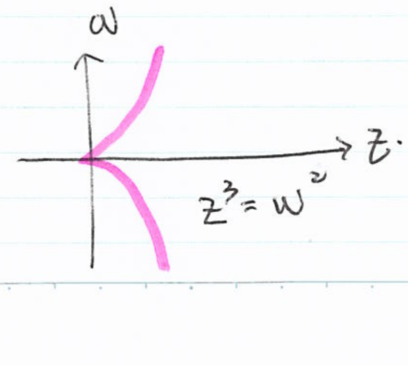
$\mathbb{C}^2 \supset C : z^p - w^q = 0 \longrightarrow U$

U_1内. $z^q - (zt)^p = 0$ … $q > p$のとき有用.

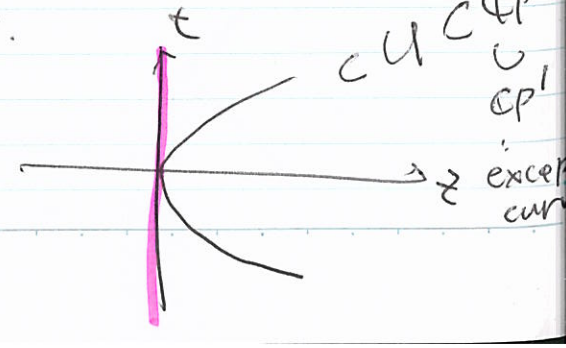
$w \to zt$ $\qquad z^p(z^{q-p} - t^p) = 0$

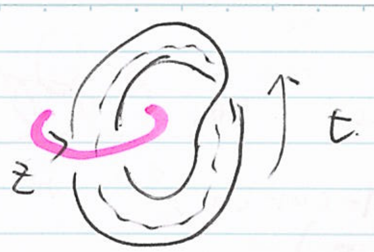
$\pi^{-1}(C)$ … U_2内. $(ws)^q - w^p = 0$ … $q < p$.

$z \to ws$ $\qquad w^q(s^q - w^{p-q}) = 0$



$z^2(z - t^2) = 0$.

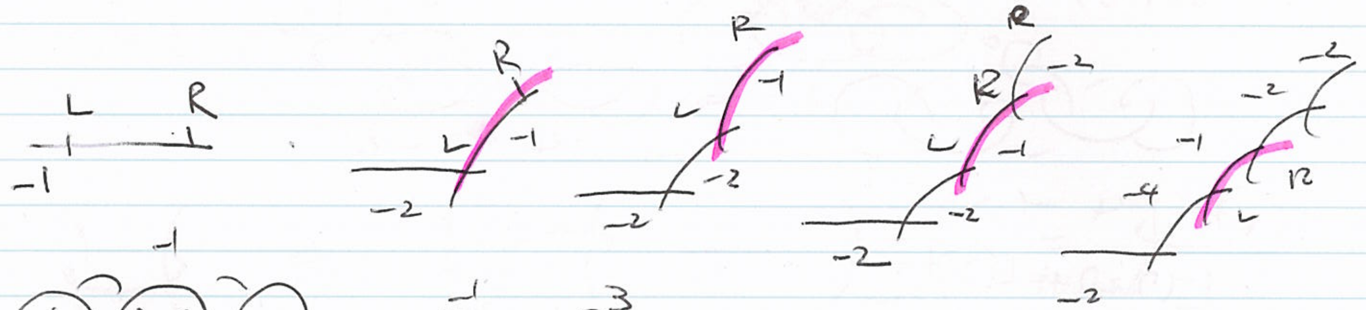


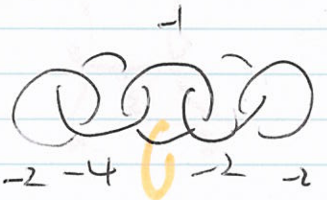


$$(p,q) \xrightarrow{p>q} (p,\ q-p)$$
$$(p,q) \xrightarrow{p>q} (p-q,\ q)$$

Euclidean algorithm.

$$(3,7) \xrightarrow[R]{} (3,4) \xrightarrow[R]{} (3,1) \xrightarrow[L]{} (2,1) \xrightarrow[L]{} (1,1)$$





$= \bigcirc^{-1} = S^3$

$K = T(3,7) : 3\times 7 + r$

r

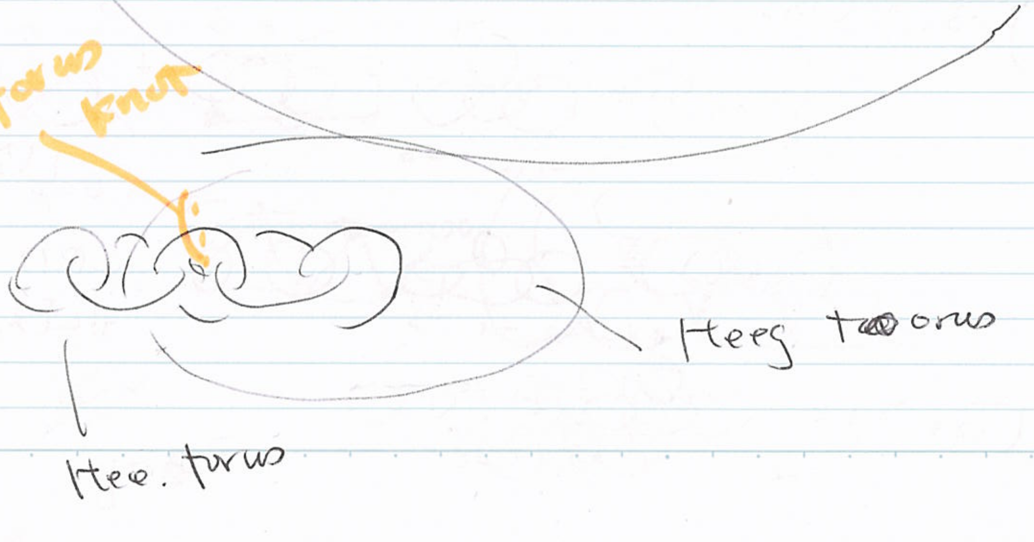
$4 - \frac{1}{2} = \frac{7}{2}$ $\quad 2 - \frac{1}{2} = \frac{3}{2}$

(p,q)から始めれば

$\dfrac{q}{q-s}$ $\quad \dfrac{p}{r}$.

$ps - qr = 1$
$0 < r < p$
$0 < s < q$.

$3s - 7r = 1$
$s = 5,\ r = 2$.
$\dfrac{7}{7-5} = \dfrac{7}{2}$, $\dfrac{3}{2}$



Lemma (1) framed link L describes S^3

(2) In the S^3 in (1), m becomes T_{pq}.

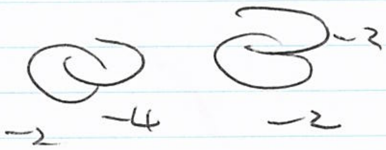
(meridian of 1-curve)

(3) r-framed m corresponds to

$(T_{pq}; pq+r)$

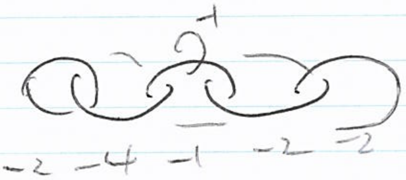
$(p,q)=(3,7)$

$M(T_{pq}; pq) = -L(p,q)\#L(q,p)$

=


$L(7,2)\#L(3,2)$

$= -L(7,3)\# -L(3,7)$



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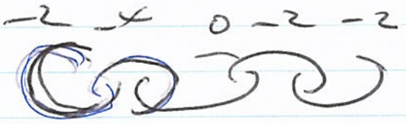

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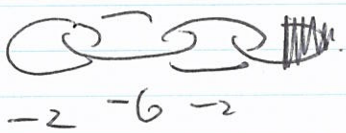
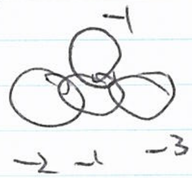


$2-\frac{1}{6-\frac{1}{2}} = \frac{20}{11}$

$M(T_{3,7}, 20)$

$L(20,11) = -L(20,3^2)$.

$= (T_{2,3}, 6+r)$

$(T_{2,3}, 5) =$


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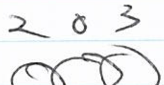
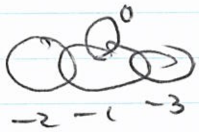
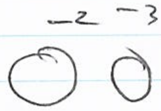


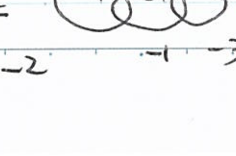
=


$L(5,1)$

$(T_{2,3}, 6) =$




$L(2,1)$
$\#L(3,1)$

$(T_{2,3}, 7) =$


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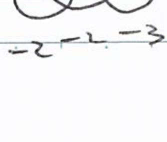
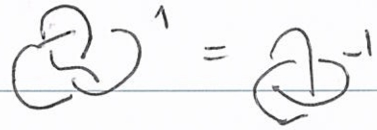
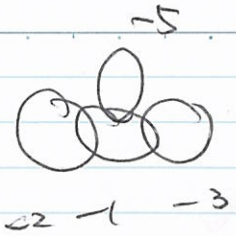






$S(-2; (2,1), (3,2), (5,4))$ Seifert

$\updownarrow$ or. rev

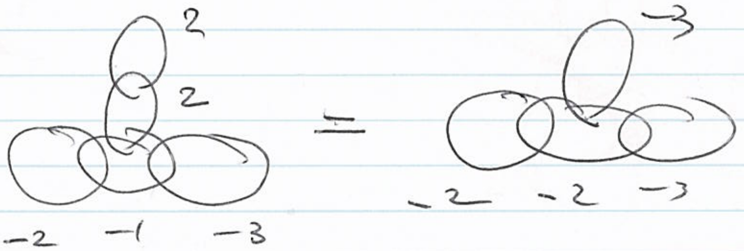
$S(2; (2,-1)(3,-2), (5,-4))$ ← Poincaré 球面

$\downarrow -3 \quad \downarrow +1 \quad \downarrow +1 \quad \downarrow +1$

$S(-1; (2,1)(3,1)(5,1))$

$(T_{2,3}, 7.5)$

$6 + 1\frac{1}{2}$



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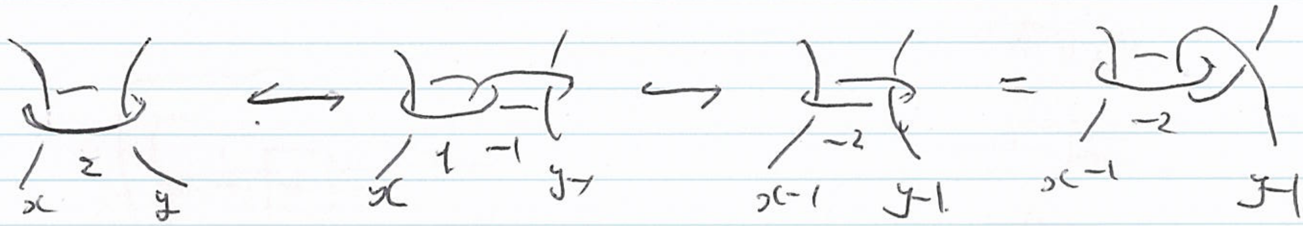
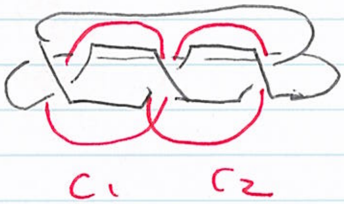
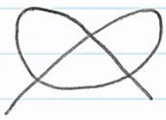


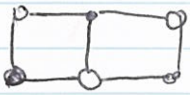
Torus knot は, fib. knot. その Monodromy の Dehn twist への分解 を Lissajous curve から求める.



Monodromy

Pr(-2,3,7)

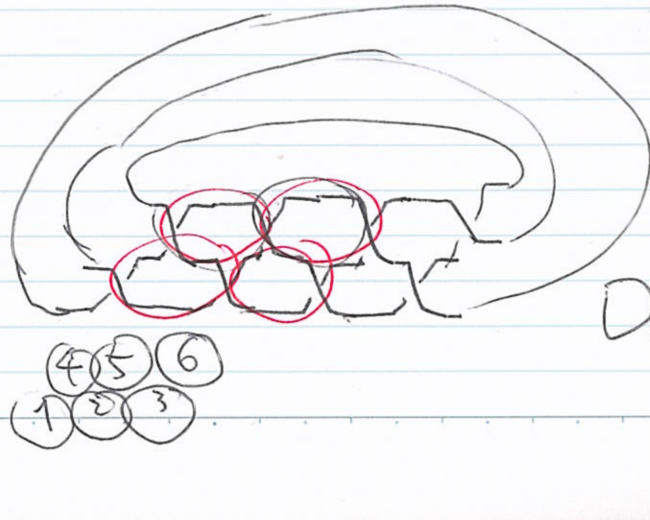




g = 3

Pr(-2,3,7)

g = 5



$D_{c_1} \sim D_{c_6}$