Cork, plug and local moves of 4-mfd's
コルク, プラグ, および4次元多様体の局所変形

X, X' exotic mfd $\Leftrightarrow$ $X \simeq X'$ (homeo)
but $X \not\cong X'$ (non diffeo)

$\simeq$ homeo

$\cong$ diffeo

Thm (Matveyev, Curtis-Freedman-Hsiang-Stong, Akbulut-Matveyev)

$\boxed{\begin{array}{l} X, X' \text{ exotic closed 4-mfds} \\ \text{, then } \exists \text{ contractible 4-mfd } C \text{ in } X, X' \\ X' \cong (X - C) \cup_\varphi C \\ \varphi : \partial C \to \partial C \quad \text{involution} \end{array}}$

__Def__ (Akbulut-Yasui)

If C: cpt contractible Stein 4-mfd

$\varphi : \partial C \to \partial C$ diffeo with $\varphi^2 = id$

φ cannot extend to $C \to C$ diffeo

, then (C, φ) is a __cork__

Cork (C, φ) is a __cork of X X'__ (exotic pair)

if $X' \cong (X - C) \cup_\varphi C$

For example

φ  W_1 (Akbulut's cork)

__Def__ (Akbulut Yasui)

If P : cpt Stein 4-mfd

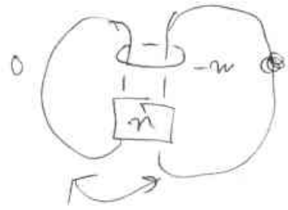
$\varphi : \partial P \to \partial P$ diffeo

φ cannot extend to $P \to P$ (homeo

, then (P, φ) is called a __plug__

plug (P, φ) is called a plug of exotic pair X, X'
if. (P, φ) is a plug and
$X' = (X - P) \cup_\varphi P$

For example

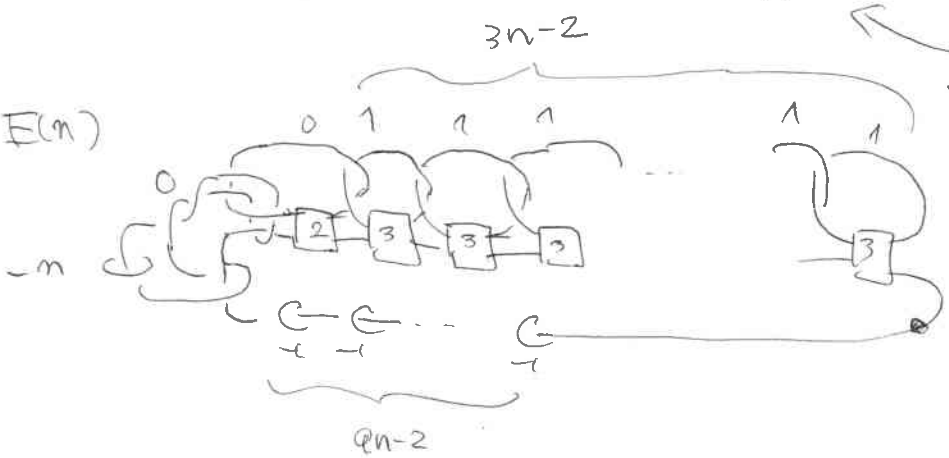


$W_{m,n}$ (plug)

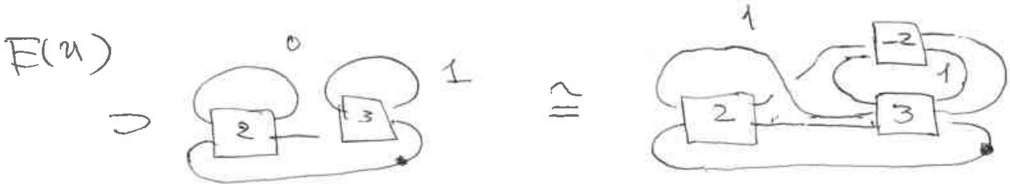
W_n

(W_1, φ) is a cork $\underset{X}{E(n) \# \overline{\mathbb{C}P}^2}$, and $\underset{X'}{(2n-1)\mathbb{C}P^2 \# (10n)\overline{\mathbb{C}P}^2}$

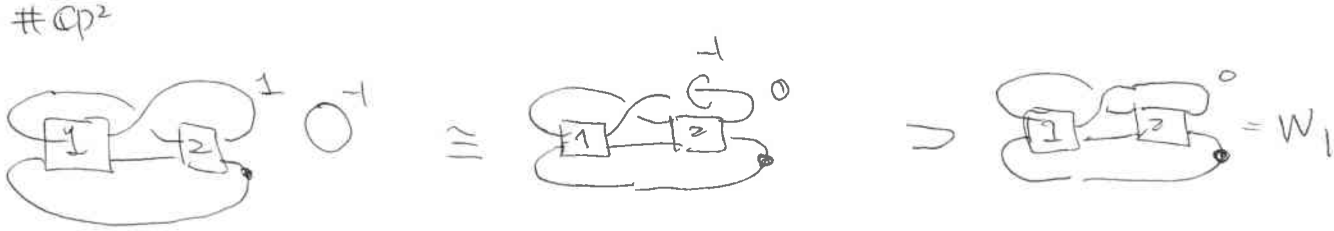
exotic pair



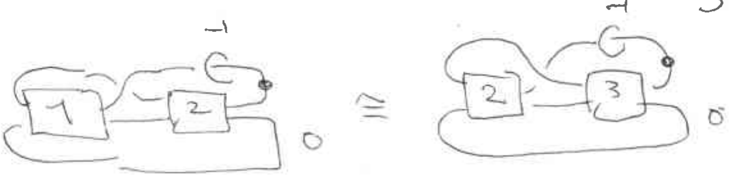
$\cup 4h$



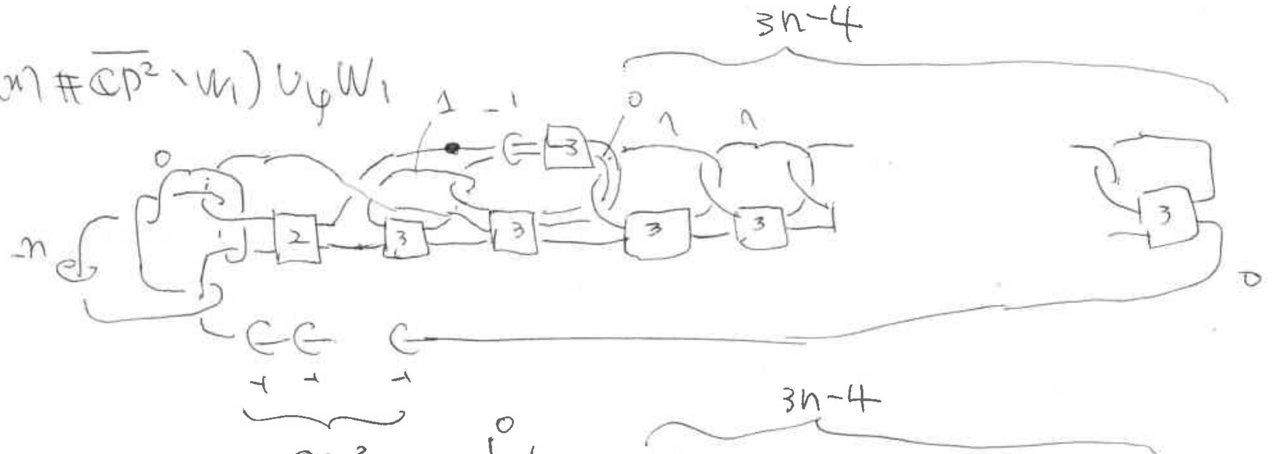
$E(n) \# \overline{\mathbb{C}P}^2$

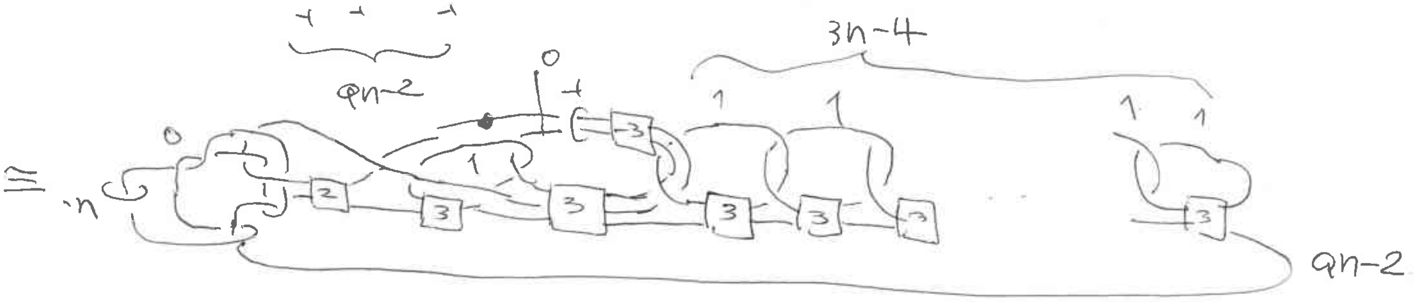


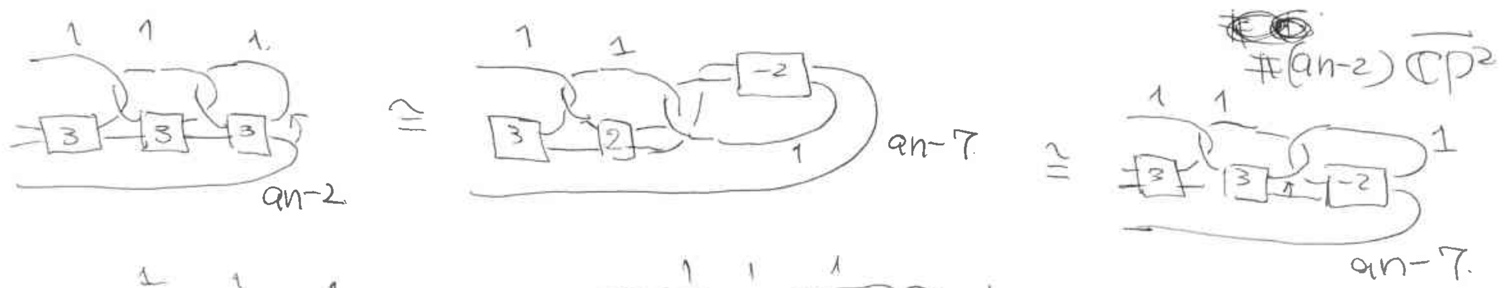
$(E(n)\#\overline{\mathbb{CP}^2}\setminus W_1)\cup_\varphi W_1 \supset$

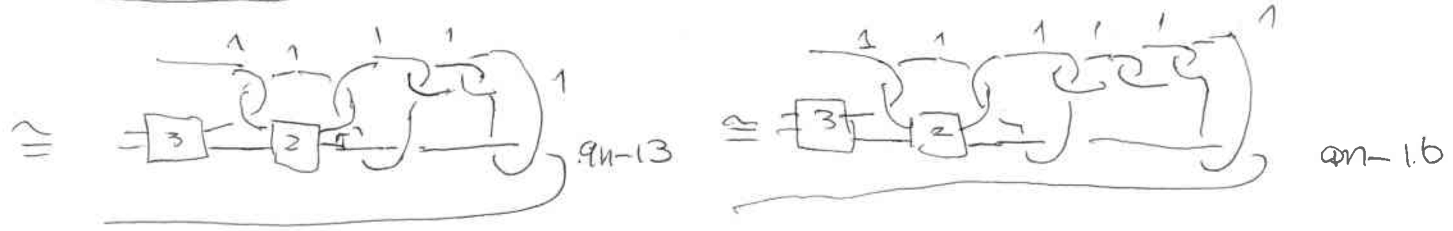


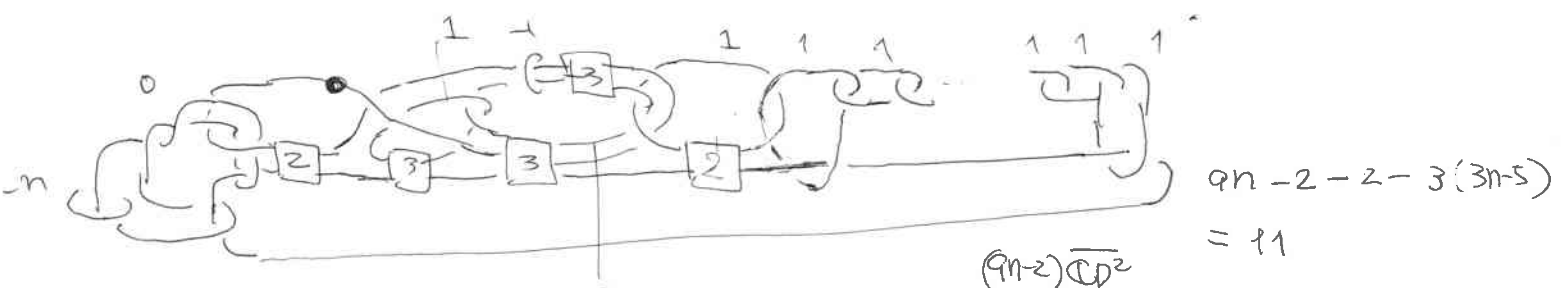
$(E(n)\#\overline{\mathbb{CP}^2}\setminus W_1)\cup_\varphi W_1$



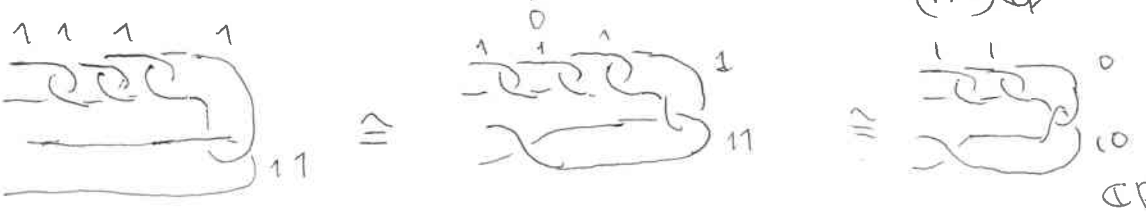




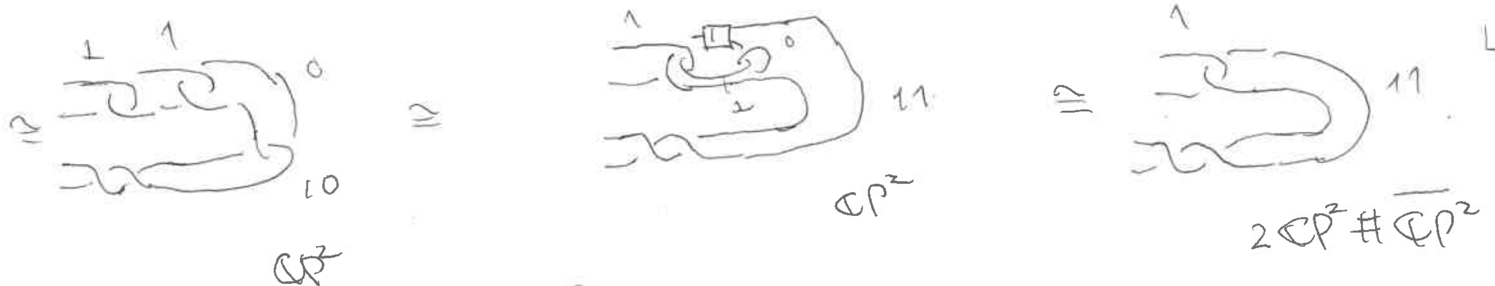


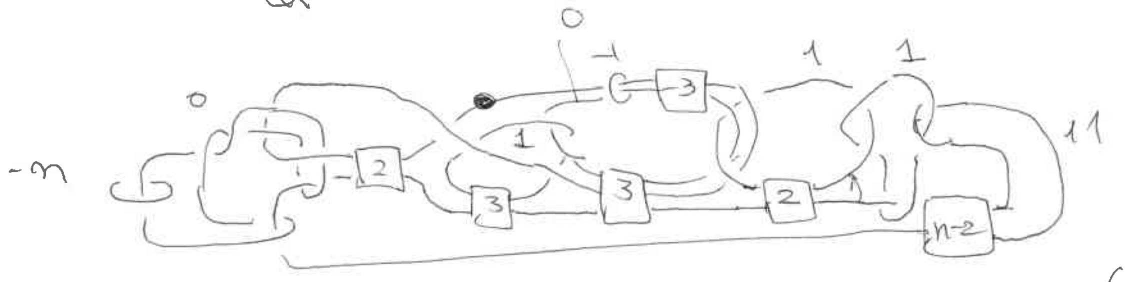


$(9n-2)\overline{\mathbb{CP}^2}$

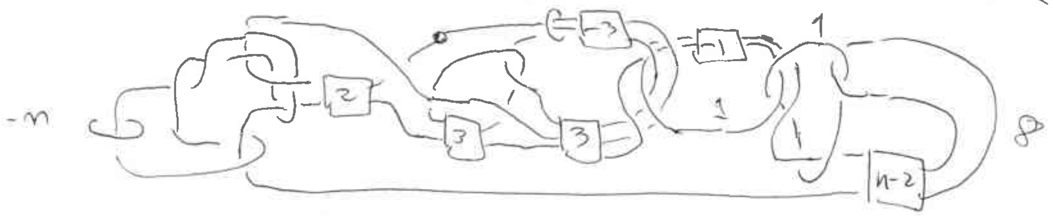


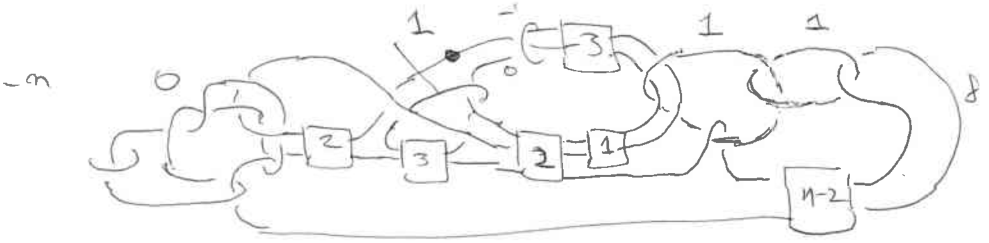
$\mathbb{CP}^2$.

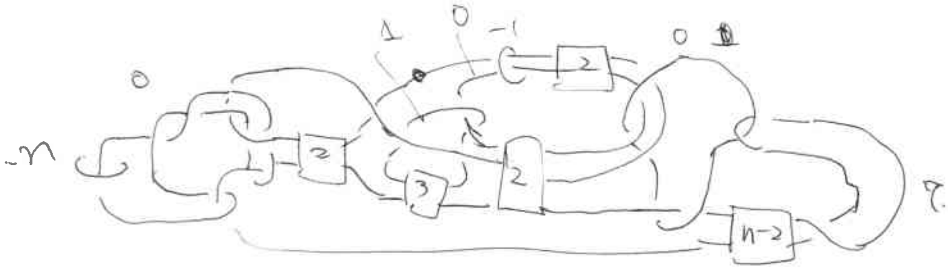




$$\# (9n-2)\overline{\mathbb{CP}^2} \;\#\; (n-2)\left(2\mathbb{CP}^2 \# \overline{\mathbb{CP}^2}\right)$$
$$= (2n-4)\,\mathbb{CP}^2 \# (10n-4)\,\overline{\mathbb{CP}^2}$$







$(2n-3)\mathbb{CP}^2 \# (10n-4)\overline{\mathbb{CP}^2}$

$2\mathbb{CP}^2 \# 4\overline{\mathbb{CP}^2}$

Thm AY
W_{2n+1} are corks of $E(2n)\#\overline{\mathbb{CP}^2}$
$W_{m,n}$ are plugs of $E(2n)\# m\overline{\mathbb{CP}^2}$
$\overline{W}_n$ are corks of $E(2n+1)\#\overline{\mathbb{CP}^2}$

Thm AY
W_1 is a cork of $E(n)_{p,q}\#\overline{\mathbb{CP}^2}$
$W_{1,3}$ is a cork of $E(n)_{p,q}\#\overline{\mathbb{CP}^2}$
W_1 is a cork of $E(n)_K\#\overline{\mathbb{CP}^2}$
$W_{1,3}$ is a plug of $E(n)_K\#\overline{\mathbb{CP}^2}$

Question What is a $\left\{\begin{array}{l}\text{plug}\\ \text{cork}\end{array}\right.$ of $E(n)_K$, $E(n)$?

Def (Cork with order p $\quad p \in \mathbb{N}_{\geq 2} \cup \{\infty\}$)

(C, φ) $\quad C$: cpt contractible Stein 4-mfd

$\varphi : \partial C \to \partial C$ diffeo $\quad$ ($\varphi^p = id$, $\varphi^i \neq id$, $1 \leq i < p$)

φ^q : $(1 \leq^{\forall} q < p)$

cannot extend to a diffeo $C \to C$

(If C is not contractible, (C, φ) is called generalized cork)

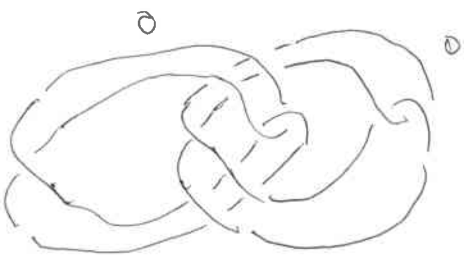
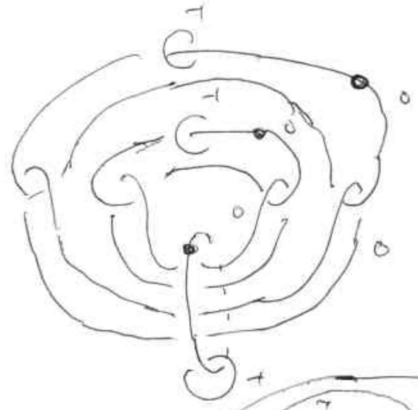
Def (Plug with order p $\quad p \in \mathbb{N}_{\geq 2} \cup \{\infty\}$)

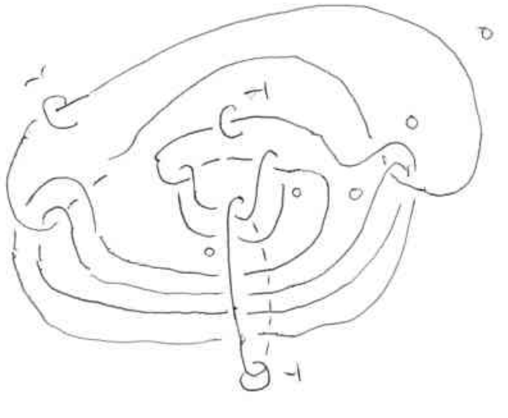
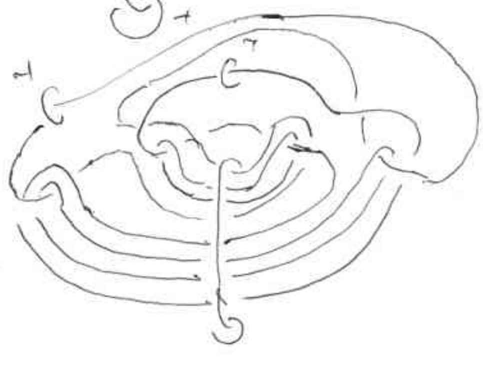
(P, φ) $\quad P$: cpt Stein 4-mfd

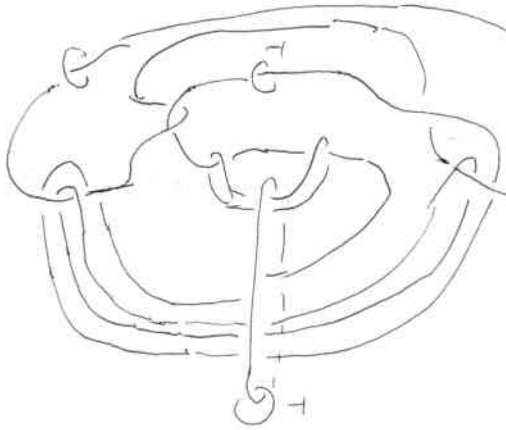
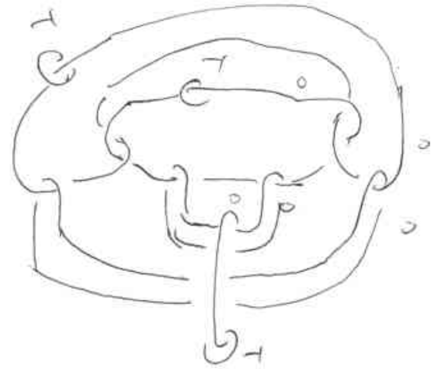
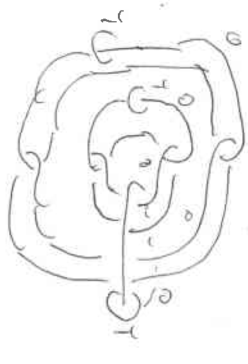
$\varphi : \partial P \to \partial P$ diffeo $\quad$ ($\varphi^p = id$, $\varphi^i \neq id$, $1 \leq i < p$)

and φ^q $\quad 1 \leq^{\forall} q < p$

cannot extend to a homeo $P \to P$

P_X  $\cong$ 

$\partial P_X \cong$  $\cong$ 

$= \partial P_X$

Let X be a 4-mfd. containg Kodaira's sing. fib type III

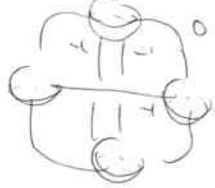
Thm (M). (P_X, φ_X) is a plug with order ∞

of X_{K_1}, X_{K_2}

, where

knot surgeries

V_{III}



$K_1 \longrightarrow K_2$

crossing change!!

$$X_{K_2} = (X_{K_1} - P_X) \cup_{\varphi_X} P_X$$

(P_X, φ_X^2) $\quad K_1 \qquad K_2$

$$X_{K_1} \longrightarrow X_{K_3}$$
$$(P_X, \varphi_X^2).$$

<u>Cor</u> (P_X, φ_X^2) is a generalized cork of X_{K_1} X_{K_3}.

another local move

K_1 $\longrightarrow$ K_4

m string.

$\rightsquigarrow (P_m, \varphi_B)$

$$X_{K_1} \xrightarrow[(P_m,\, \varphi_B)]{} X_{K_4}$$

plug twist

, where X admits some singular fib.

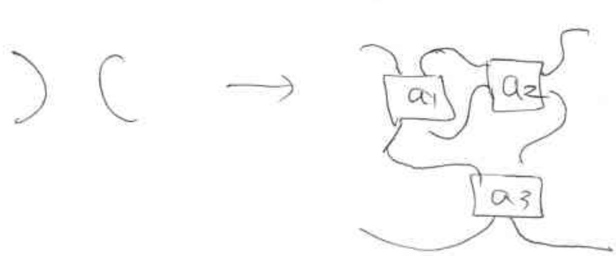
Thm (M). $\vec{S} = (a_1 a_2 \cdots a_{2n+1})$.

$a_1 \equiv a_2 \equiv \cdots \equiv 0 \, (2)$

$\exists (P_X, \varphi_{\vec{S}})$ a plug with order ∞

$$X_{K_1} \longrightarrow X_{K_2}$$

$$K_1 \longrightarrow K_2$$

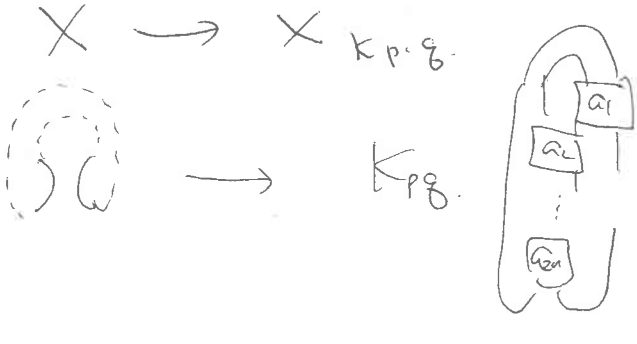


etc.

Cor $K_{p.q.}$ 2-bridge knot

$\exists (P_X, \varphi_{pq})$

$$X \longrightarrow X_{K_{p.q.}}$$

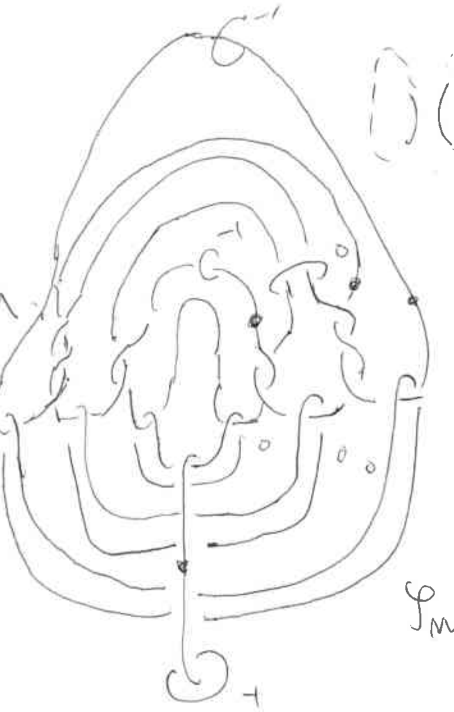


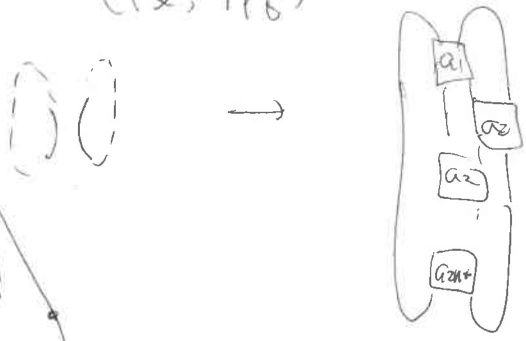
Cor 2 K_{pq} 2-bridge link

$X \supset C \# C$ ← cusp fib.

$$X \xrightarrow[(P_X, \varphi_{pq})]{} X_{K_{pq}}$$

M





$= K_{pq}$

$\partial M = \partial P_X$

$\varphi_M : \partial M \to \partial M$ involution

Let X be as above.

$$X_{K_1} \xrightarrow[(M, \varphi_M)]{} X_{K_2}$$

K_1 K_2

mutant

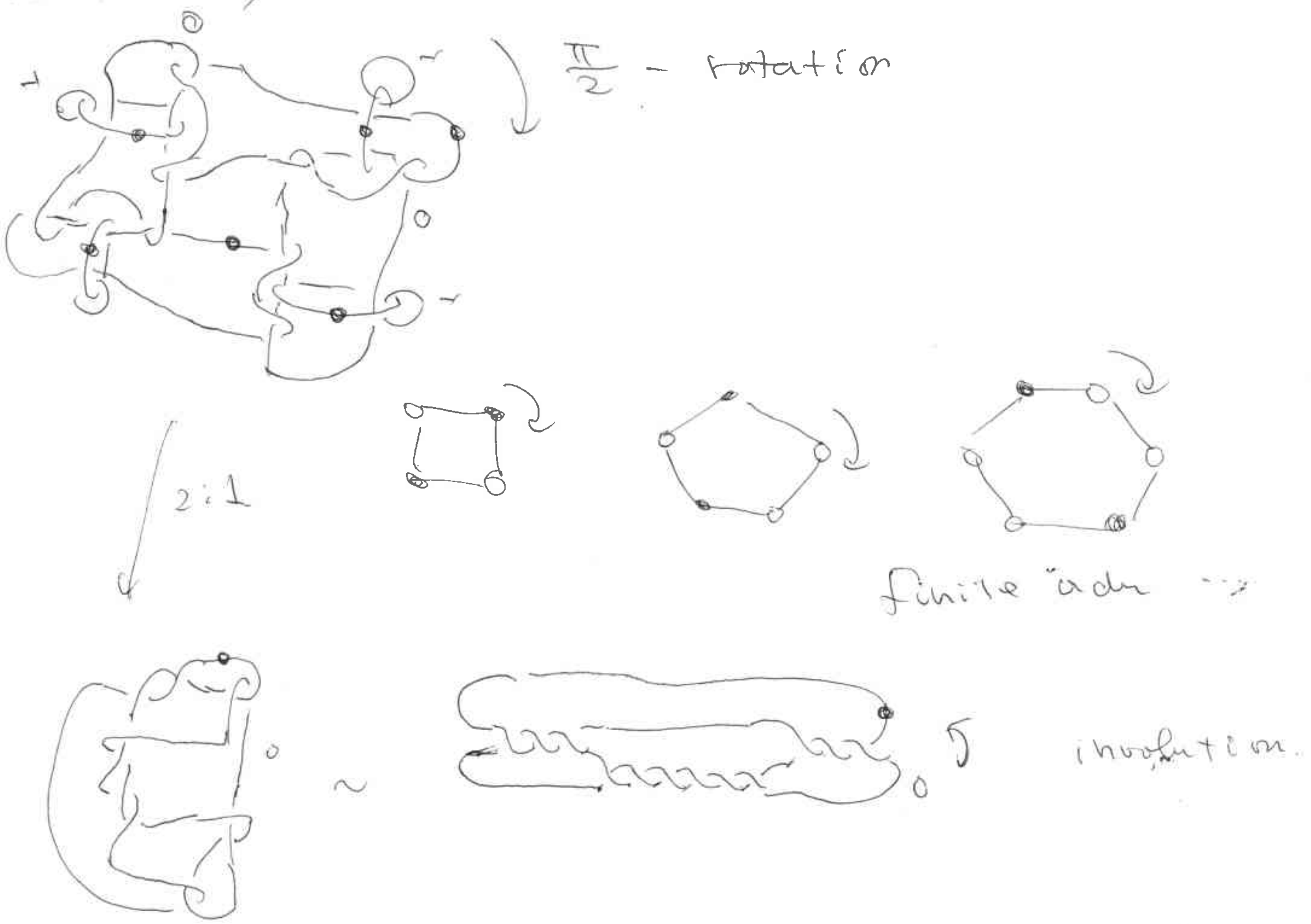
Question. (M, φ_M) is a plug or not?

M is not Stein.

If φ_M extends to a $M \to M$ diffeo, then $K_1 \sim K_2$ mutant

$\Rightarrow X_{K_1} \cong X_{K_2}$.

Furthermore, ...



<u>Problem</u> Construct new type of cork and plug.