ハンドルセミナー　　　　2013/6/26

No.1

$\varepsilon \in \mathbb{C}$: fixed.

$$M(p,q,r) := \{ (x,y,z) \in \mathbb{C}^3 \mid \underbrace{x^p + y^q + z^r}_{f(x,y,z)} = \varepsilon \}$$ Milnor fiber

$$\Sigma(p,q,r) := \{ \quad '' \quad \mid \quad '' \quad , |x|^2+|y|^2+|z|^2 = 1 \}$$

$$S^5 \setminus \Sigma(p,q,r) \xrightarrow[\pi]{\text{Milnor map}} S^1 , \quad \partial M(p,q,r) = \Sigma(p,q,r)$$

$\pi = \dfrac{f}{|f|}$

$\mathrel{\text{S}\!\shortparallel}$ $M(p,q,r)$ - fiber

$$\begin{array}{ccccc} M(p,q,r) & \xrightarrow[r:1]{\varphi} & D^4 & \subset & \mathbb{C}^2 \\ \Psi & & \Psi & & \\ (x,y,z) & \longmapsto & (x,y) & & \\ & & \cup & & \\ & & B = \{ x^p + y^q = \varepsilon \} & & \text{locus} \end{array}$$

$$M(p,q,r) \setminus \varphi^{-1}(B) \xrightarrow[r\text{-covering}]{} D^4 - B$$

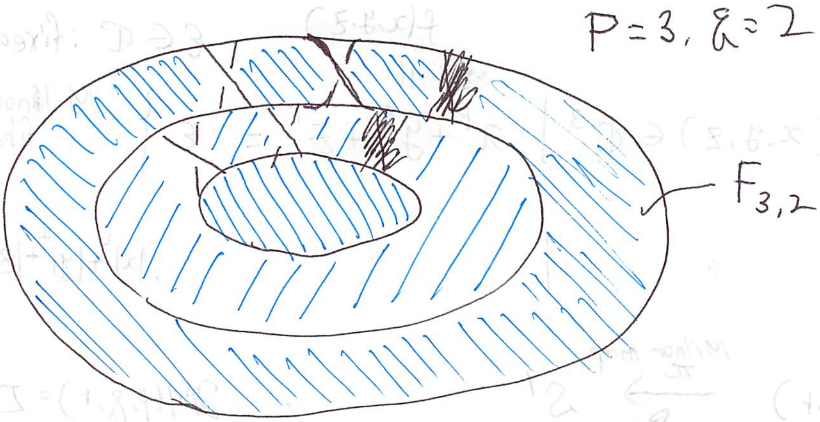
$$\begin{array}{ccc} S^3 & \subset & D^4 \\ \cup & & \cup \\ B \cap S^3 & & B \\ \| & & \\ T_{p,q} & : & (p,q) \text{ torus link} \end{array}$$

$$S^3 \setminus T_{p,q} \to S^1$$
$\uparrow$
$F_{p,q}$: $\exists$ minimal genus $T_{p,q}$ の Seifert surface.

$\{ B \cong F_{p,q}$ となる $\}$

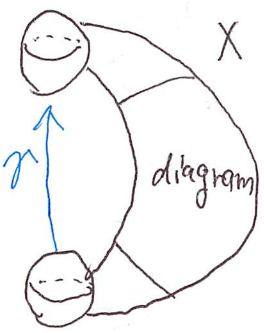
No.2

$P=3,\ q=2$



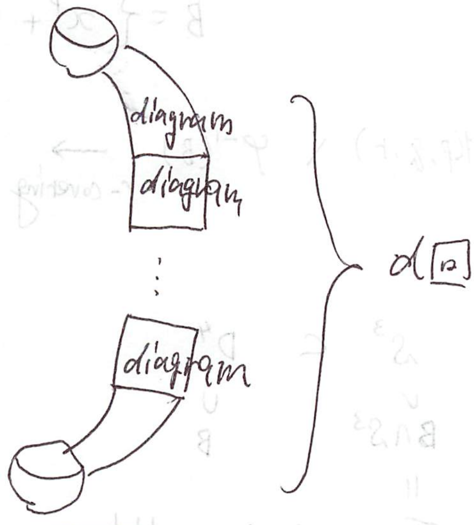
---

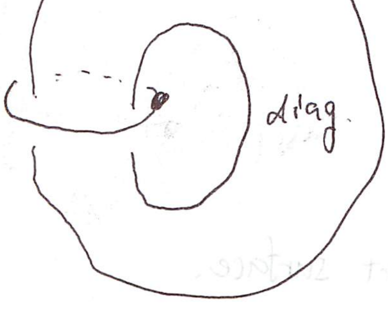
cyclic Covering のハンドル分解

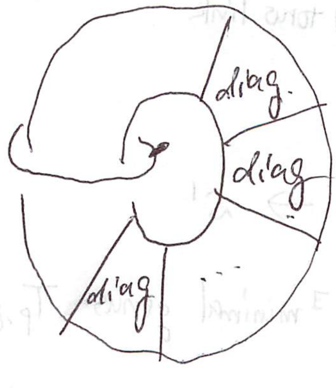


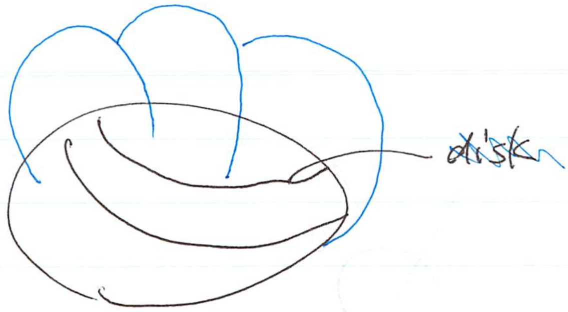
$$\pi_1(X) \to \mathbb{Z}_d$$
$$\gamma \mapsto 1$$
$$\tilde{X} \to X$$

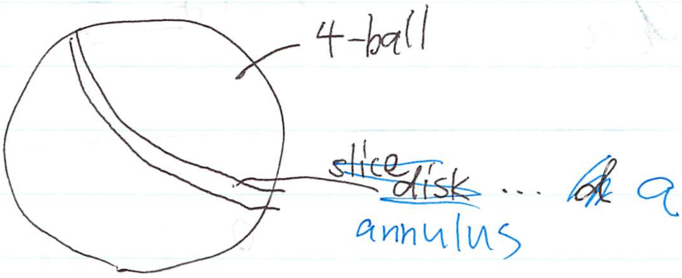
$\tilde{X}$





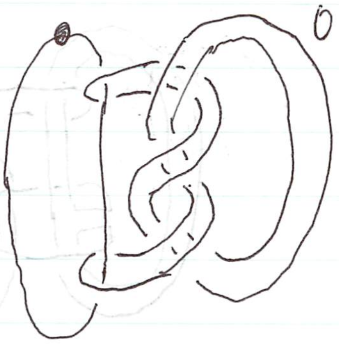






モーションピクチャー

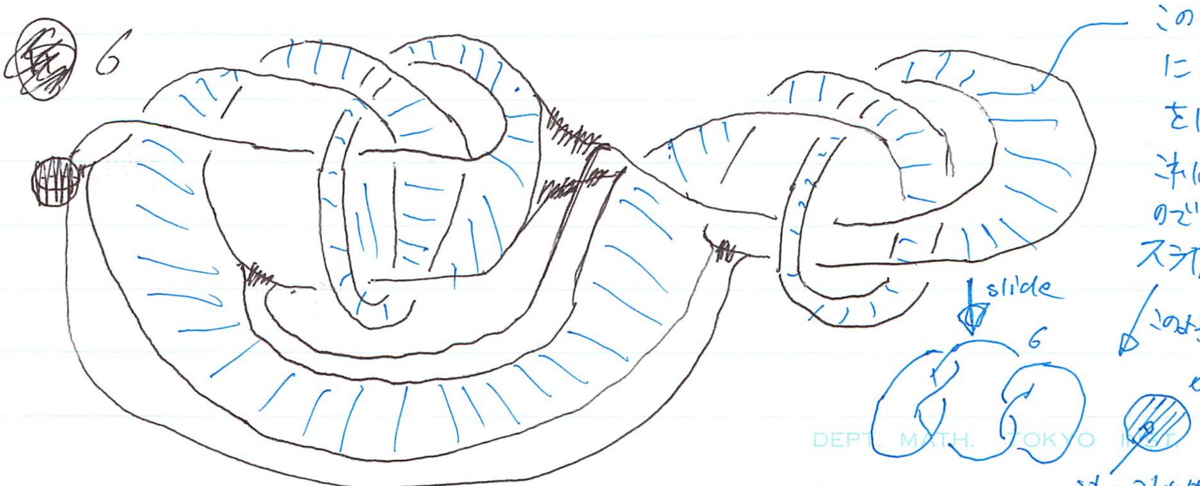
$D^4 - a =$



disk でなく annulus
なってるん いらない

annulus exterior

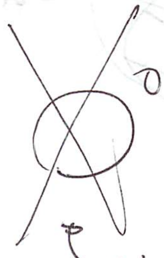
6

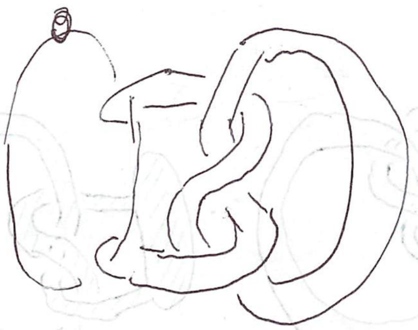


このリボンに3hをはる。
これはみにくいのでハンドルスライドすると
このようになる
となる
これに3hをはるのでcancelする

No. 4

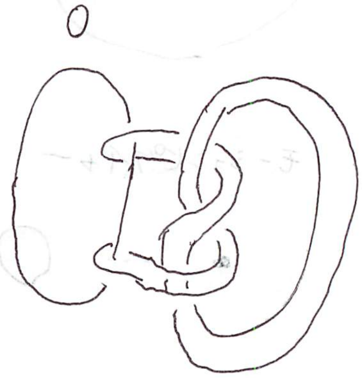
$\tilde{p} \longrightarrow D^4 \supset a$



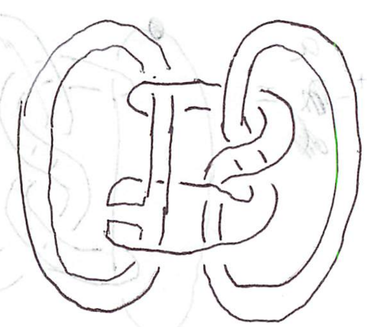


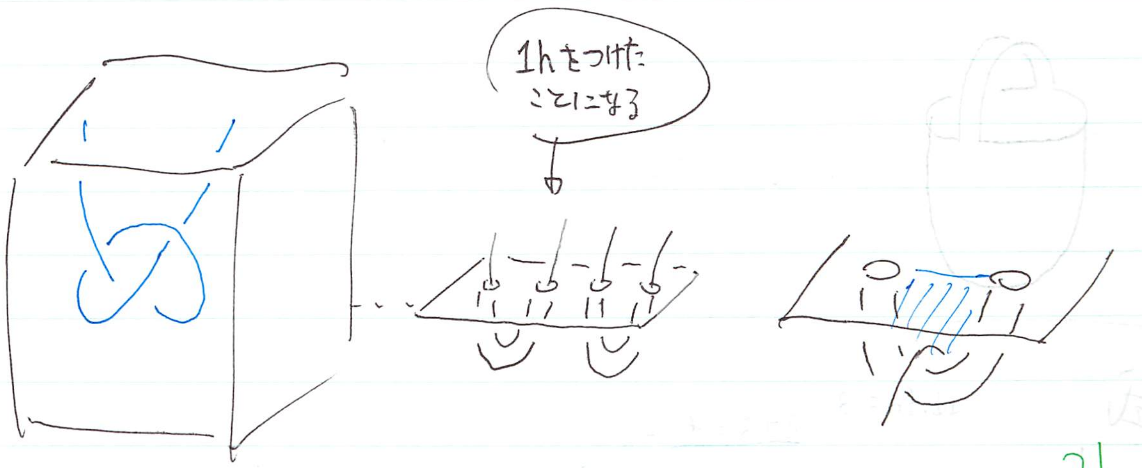
の境界を知りたい。

$\xrightarrow{\text{dot を 0 に}}$



↓ slide

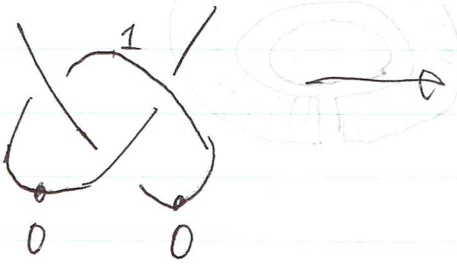


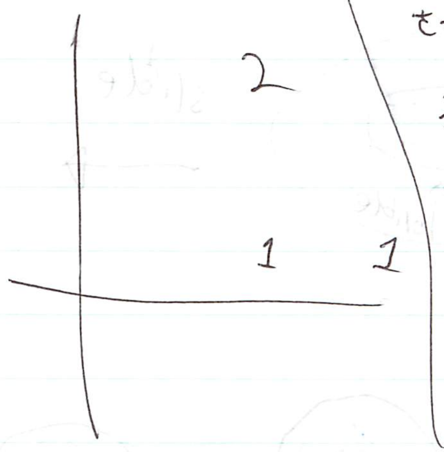


Bathtub \ $\mathring{N}(K)$
=
水

(Kの0h があると
↓
complement に1hつける)

+1すればよい



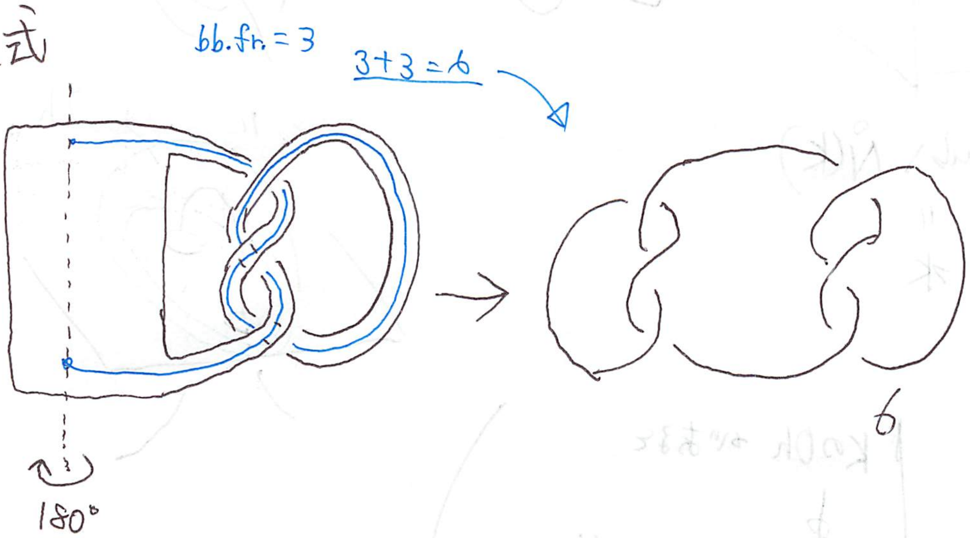


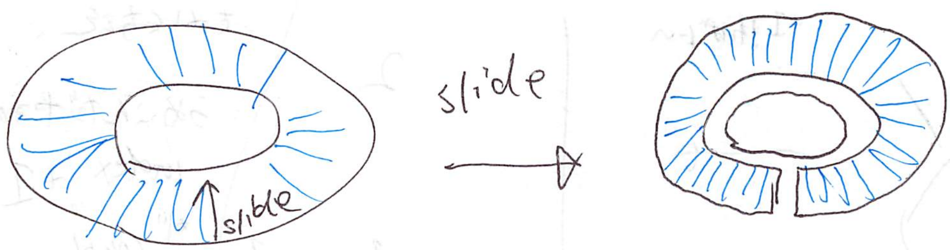
complement
のhandle分解
をかく方法、

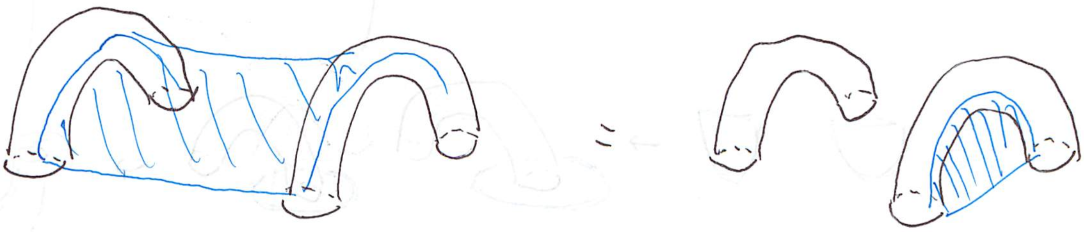
うめこんだやつの
index +1
が complement
の index になる

No.6

公式



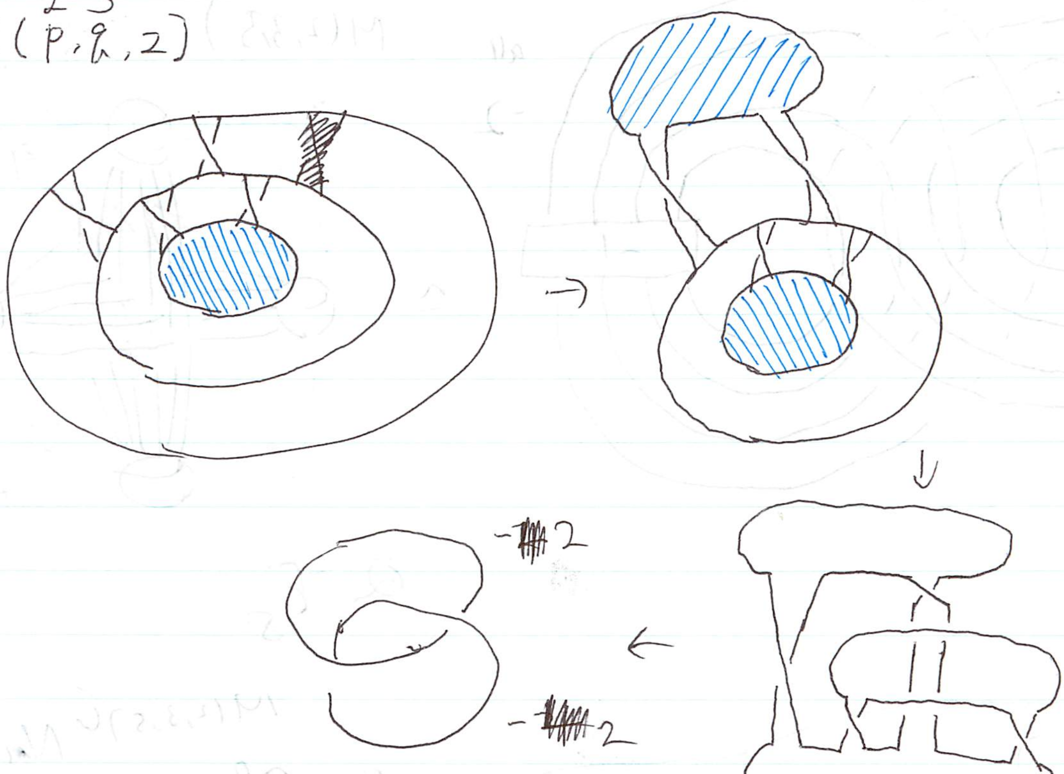




2013/6/26

No. 7

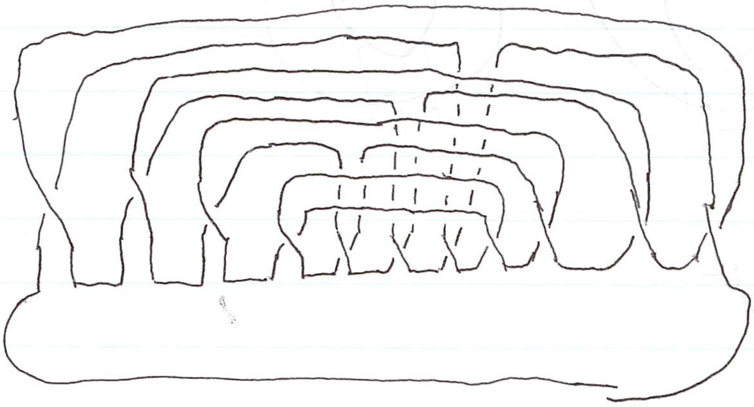
$M(\overset{2}{p}, \overset{3}{q}, 2)$



$\Sigma(2,3,2)$

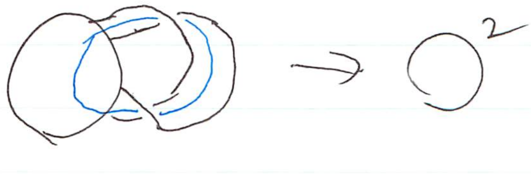
$$-2 \cdot \frac{1}{2} = -2 + \frac{1}{2} = -\frac{3}{2}$$

$M(3,5,2)$



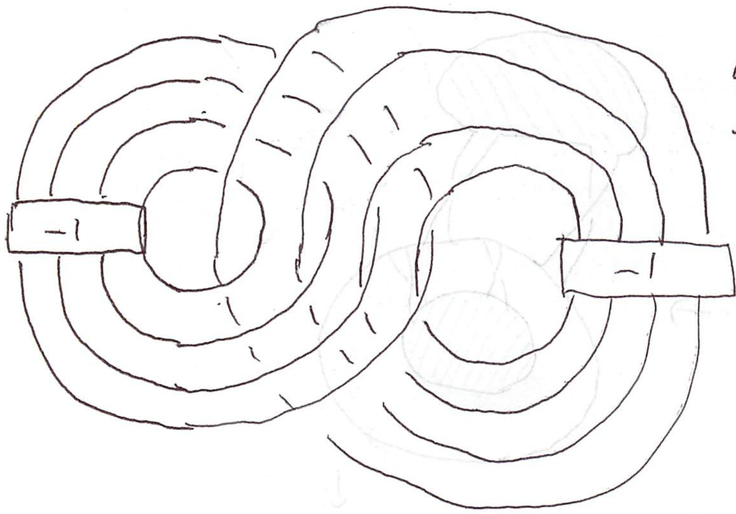
$M(1,2,2) \quad \phi = S^3$

$M(2,2,2)$



No. 8

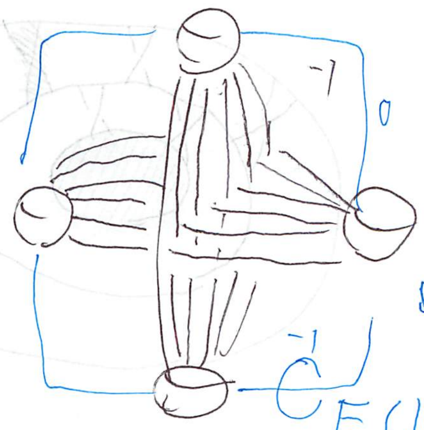
$b_2(M(p,q,r)) = (p-1)(q-1)(r-1)$



all -2

$M(2,3,5)$

$\simeq$



$Q = E_8$

$M(2,3,5) \cup N_1 = E(1)$

$\cong \mathbb{CP}^2 \# 9 \overline{\mathbb{CP}^2}$

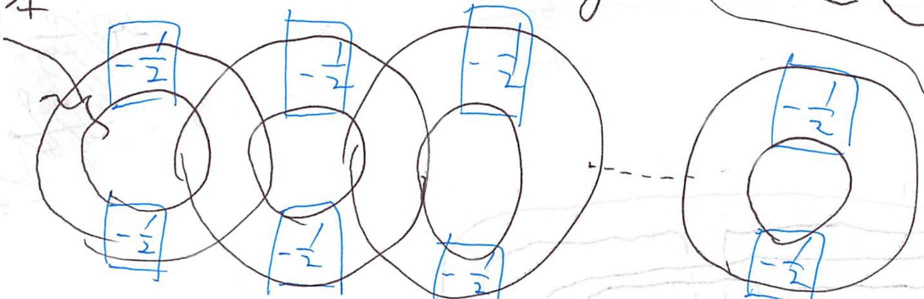
N_1

$-E_8 \oplus H = (1) \oplus^9 (-1)$

$M(p,q,2)$

$q-1$個のring

$p-1$本



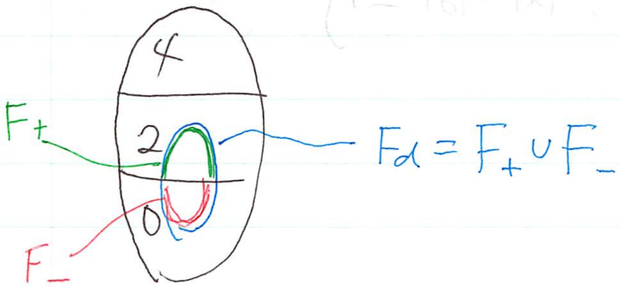
all $= -2$

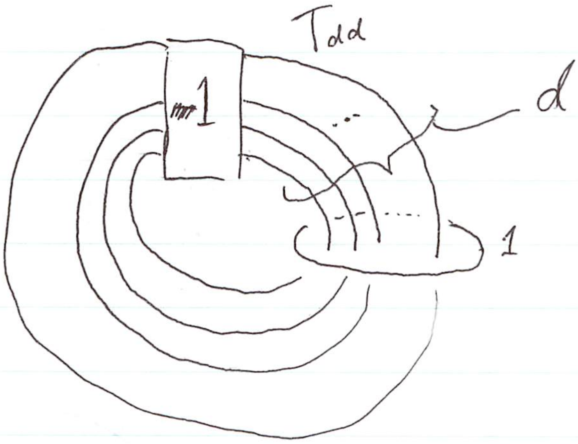
$M(p,q,r) \to D^4$

$M(2,3,6n-1) \cup N_n = E(n) = \#_5^n E(1)$

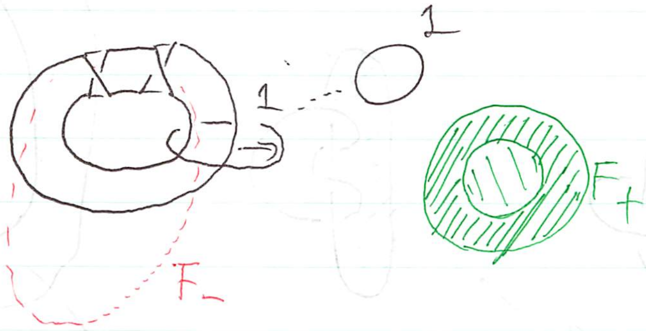
$\mathbb{CP}^2 \supset F_d$: holo curve with degree d

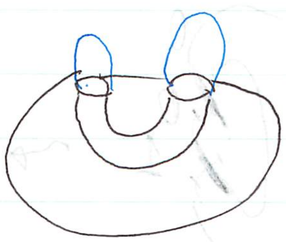
$F_d = \{ (x, y, z) \mid x^d + y^d + z^d = 0 \}$



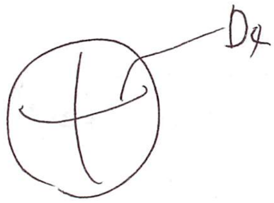


${}_nC_2$
$=$
$\frac{1}{2} d(d-1)$ の交点





No.10

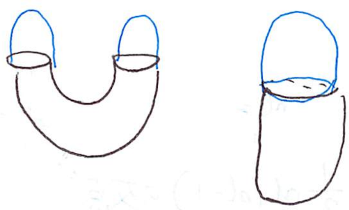


$$\mathbb{C}^2 \supset D^4 \supset S^3 \supset H = \{ xy=0,\ |x|^2+|y|^2=1 \}$$

Thm

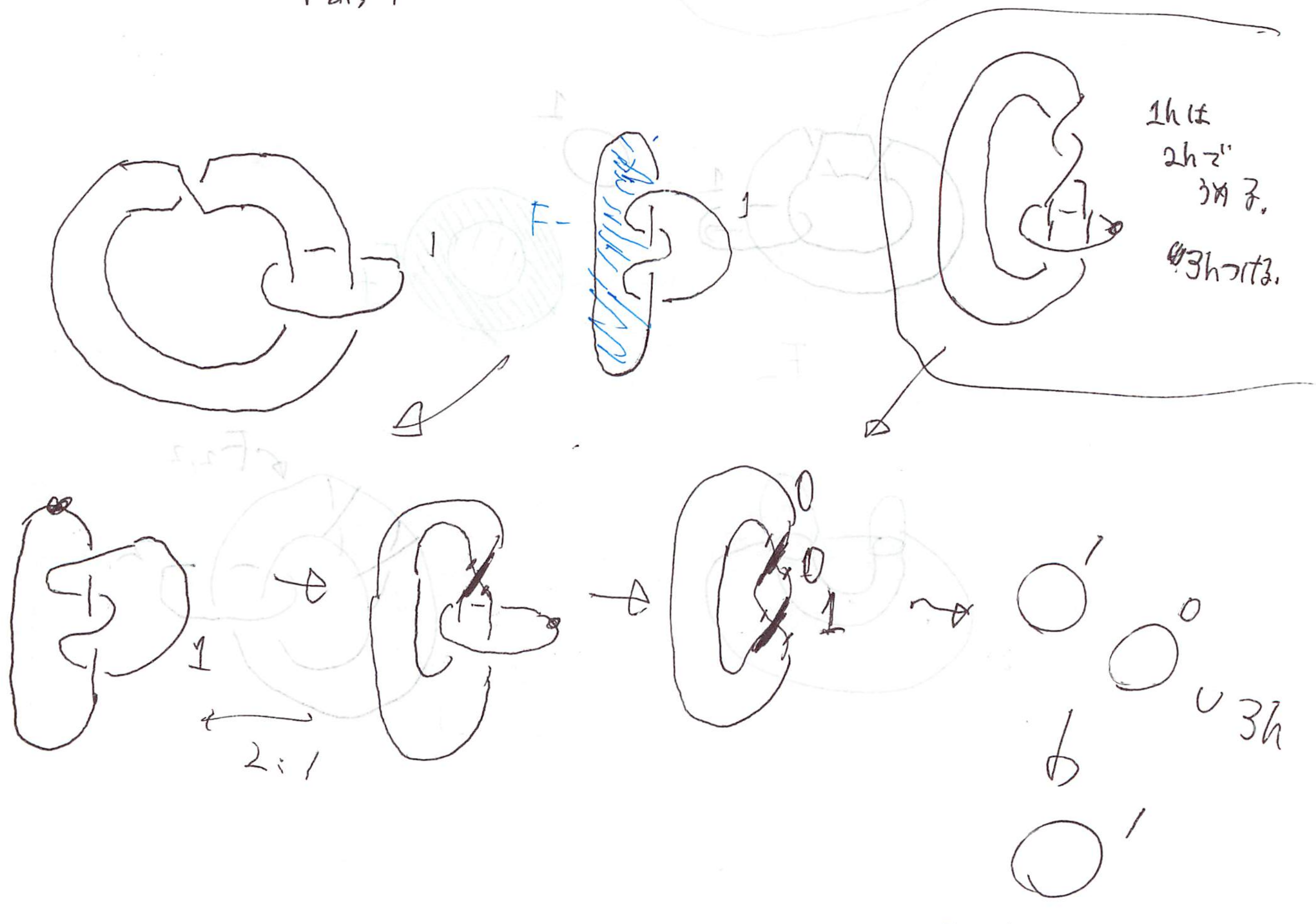
$F_d = F_{d,d} \cup$ d個のdisks

F_d



$\|$

$F_{d,d-1} + \underbrace{\text{1個のdisk}}_{\substack{\| \\ F_{d,-1}}}$



$\mathbb{C}P^2 \to \mathbb{C}P^2$

$(1\ -1)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix}$

$$\mathbb{C}P^1 \times \mathbb{C}P^1 \curvearrowright \mathbb{Z}/2 \quad \supset \Delta$$

$\Delta \cdot \Delta = 2$

$\bar{\Delta} \cdot \bar{\Delta} = -2$

$\bar{\Delta} = \{(x, -\frac{1}{\bar{x}}) \mid x \in \mathbb{C}P^1\}$

$$\downarrow$$

$$\mathbb{C}P^1 \times \mathbb{C}P^1 / (x,y) \sim (y,x) = \mathbb{C}P^2 \supset \mathbb{R}P^2$$

$\Delta \downarrow F_2$ br. locus

$F_2 \cdot F_2 = 4$

$\mathbb{R}P^2 \cdot \mathbb{R}P^2 = -1$

$$X(n,m) \xrightarrow{2:1} \mathbb{C}P^1 \times \mathbb{C}P^1$$

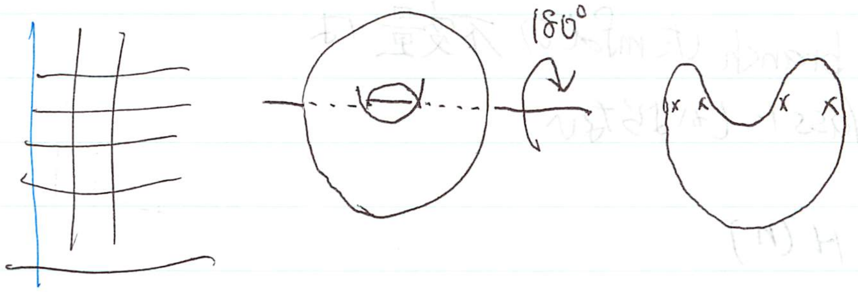
$$\cup$$

$$B_{n,m} = 2n[S^2 \times pt.] + 2m[pt \times S^2]$$

$X(n,2) = E(n)$

$X(1,2) \fallingdotseq E(1)$

$X(1,2)$ が $E(1)$ になること：

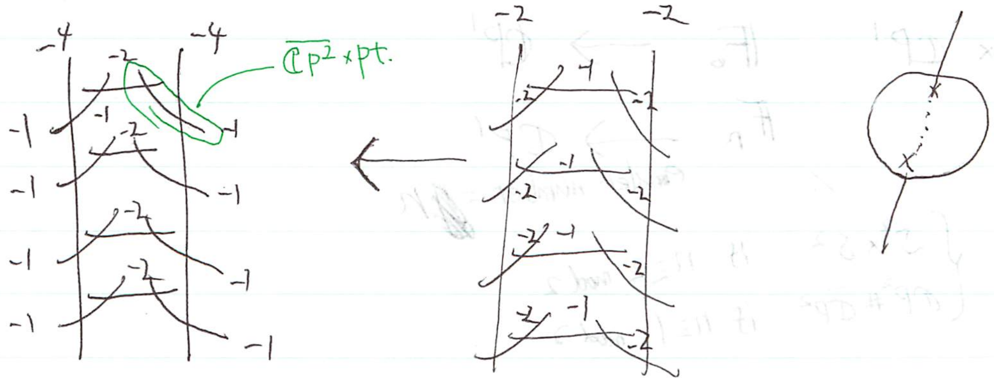


$\mathcal{L}^2 \to \mathcal{L}$

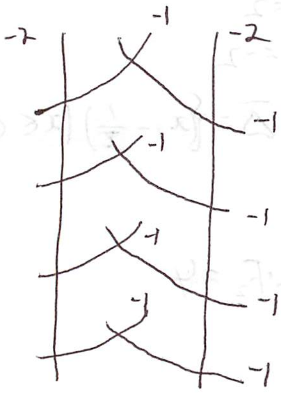
$\downarrow$

$S^2 \to S^2$

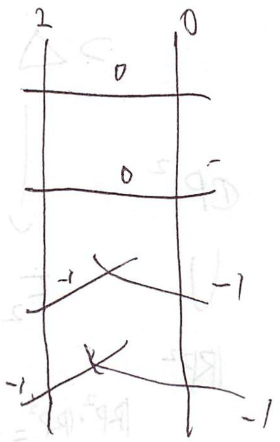
$z \to z2$



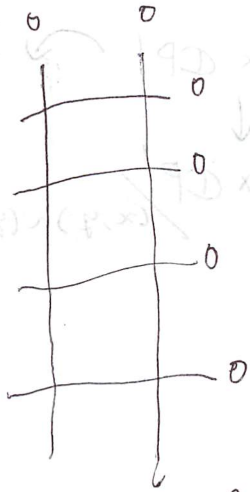
No. 12



$4\overline{\mathbb{C}P^2}$



$6\overline{\mathbb{C}P^2}$

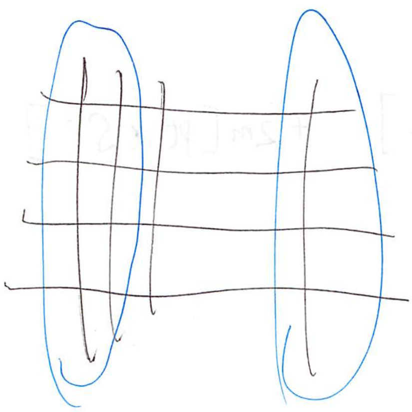


$S^2 \times S^2$

$8\overline{\mathbb{C}P^2} \# S^2 \times S^2$

$\parallel$

$\mathbb{C}P^2 \# 9\overline{\mathbb{C}P^2}$



$X(1,2) = E(1)$

$X(n,2) = E(n)$

$B_{m,n}$ で branch した mfd の不変量は homology class にしかよらない.

$H(2n)$ $\quad H'(n)$

$\mathbb{C}P^1 \times \mathbb{C}P^1 \quad \mathbb{F}_0 \longrightarrow \mathbb{C}P^1$

$\mathbb{F}_n \longrightarrow \mathbb{C}P^1$

euler number $= n$

$$\parallel$$

$$\begin{cases} S^2 \times S^2 & \text{if } n \equiv 2 \bmod 2 \\ \mathbb{C}P^2 \# \overline{\mathbb{C}P^2} & \text{if } n \equiv 1 \bmod 2 \end{cases}$$

2013/6/26

No. 13

$H(2n) \cong H'(n)$ homeo

変形同値でない

◎ $H(2n) \underset{\text{diffeo}}{\approx} H'(n)$ か？

minimal

-1 curve を含む

$n=1$

$H(2) = E(3) \not\approx H'(1)$

$n=1$ の時は exotic. 一般の場合は分からない。

Cork

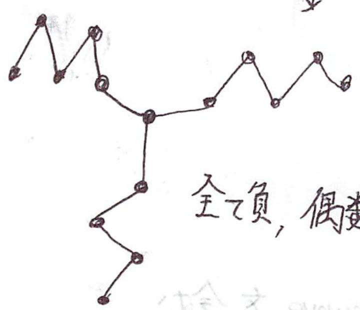
$B^2 \subset D^4$

D^4 の中のリボンを branch して cork をつくった。と Akbulut がいってた

これだと思われる。

No.14

$\Sigma(p, q, r)$ plumbing diagram

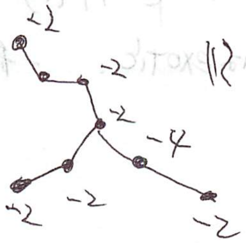


全て負, 偶数

$\cong \Rightarrow -E_8$

$(2,5,3)$

$(3,4,7)$



$\Sigma(a, b, c)$

$\Sigma(p^n, q^n, r^n)$

$$\lambda = \frac{1}{8}\sigma = \frac{-b_2}{8}$$