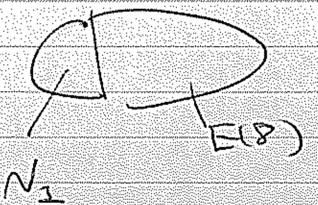
$\Sigma(2,3,5) \hookrightarrow \mathbb{CP}^2 \# 9\overline{\mathbb{CP}^2}$



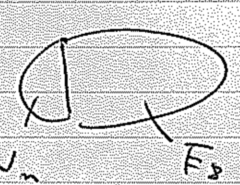

$\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \oplus E_8$

Actually

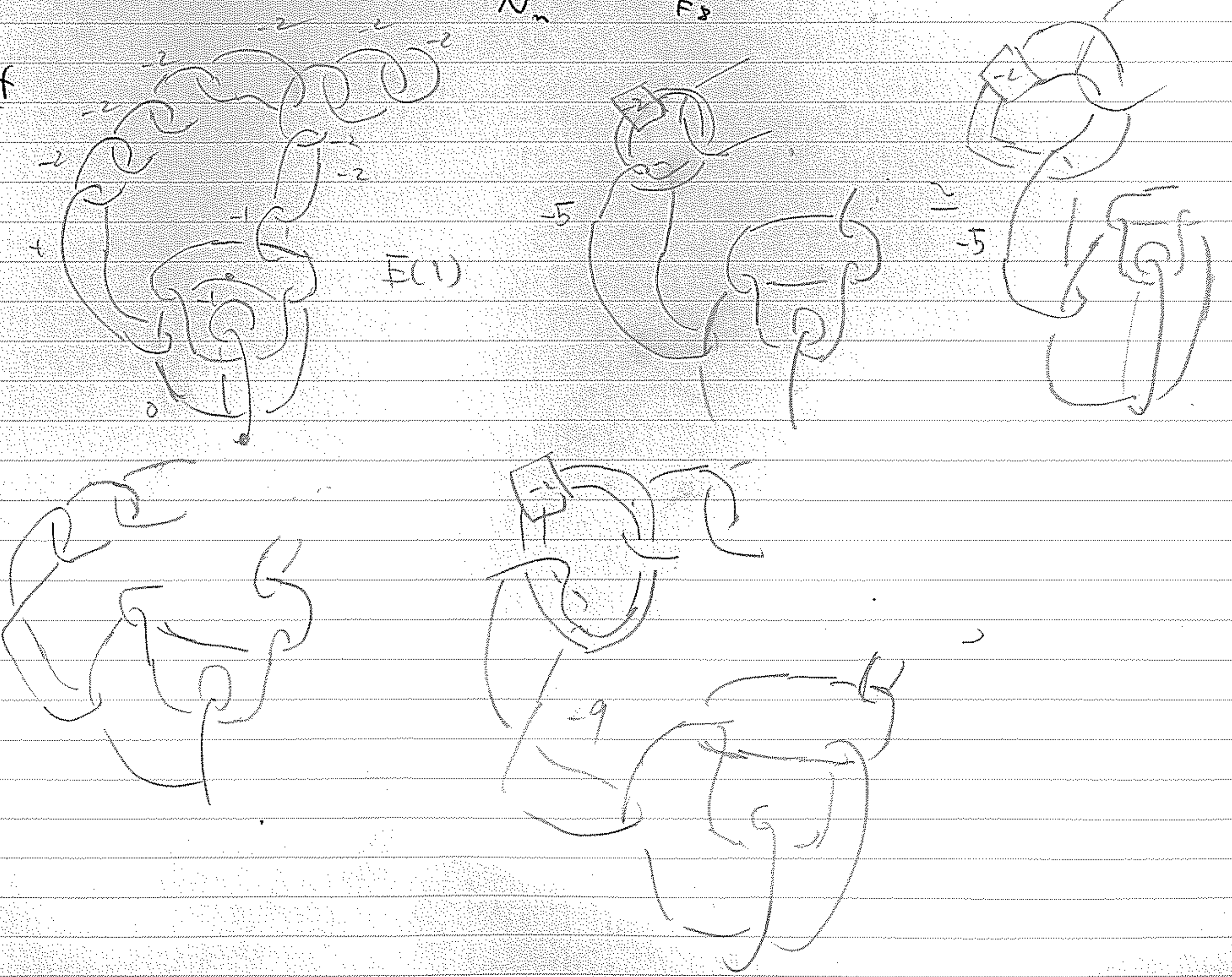
$\Sigma(2,3,6n-1) \hookrightarrow \mathbb{CP}^2 \# 9\overline{\mathbb{CP}^2}$

$(1 \leq n \leq \cancel{27})$
33
29

$n$: odd




Proof



$$\begin{pmatrix} 0 & 1 \\ 1 & -33 \end{pmatrix} \oplus E_8 \qquad -(40-5\cdot(-8)-8) = -72.$$

$$33 = 2n+1$$

$$x^2 = -\cancel{33}\,29 = -n \quad \text{odd}$$

$$t^2 = 0$$

$$x = \left(-3h + \sum_{i=1}^{9} e_i\right) n + h_1 = nt + e_1$$

$$x^2 = n^2 \cdot t^2 + 2n\, t \cdot e_1 + e_1^2$$
$$= -2n - 1 = -\cancel{33}\,29$$

$$x = nt + e_1$$

c.f. This class representing sphere.

$x$: non characteristic

~~M.J. D. Hamilton~~ $\quad \Sigma(2,3,6n-1) = \partial E_8$
$$1 \le n \le \cancel{33}\,29$$

$$d(\Sigma(2,3,6n-1)) = 2$$

$$\therefore \quad c^2 + b_2 \le 8$$

**Question** $\quad \forall n$: odd $\quad \Sigma(2,3,6n-1)$ bounds $E_8$ mfd?

M J Hamilton 1 $\quad \pi_1^X = e \qquad A \hookrightarrow X^4$

$$A^2 = -N < 0,$$

If. $\quad A = g A' \qquad g$ : prime power.

$$\Rightarrow N \le 2(b_2 - \sigma) + 4 g_A \qquad (g = 2)$$

$$N \le \frac{2g^2}{g^2-1}(b_2 - \sigma) + \frac{4g^2}{g^2-1} g_A \qquad g \ne 2$$

" 2 $\quad X^4 \quad \pi_1 = e \qquad A \hookrightarrow X \qquad A^2 = -N < 0$

$A$: char.

$$\Rightarrow N \le 4b_2 - 5\sigma - 8 + 8 g_A$$

Thm (R.) $X_m = \mathbb{C}P^2 \# m \overline{\mathbb{C}P^2}$ $\Sigma$. $\Sigma^2 \geq 0$. $K_X = -3S_0 - \sum S_i$

$$2g-2 \geq K_X \cdot |\Sigma| - \Sigma^2$$

$$g \geq \frac{1}{2}\left(K_X \cdot |\Sigma| - \Sigma^2\right) = 1 + \frac{1}{2} K_X \cdot |\Sigma| - \frac{1}{2}\Sigma^2$$

$$\Sigma = a_0 S_0 + \sum_{i=1}^{m} a_i S_i$$

$$\Sigma^2 = a_0^2 - \sum a_i^2$$

$$\frac{1}{2} K_X \cdot |\Sigma| = \frac{1}{2} K_X \cdot \left(a_0 S_0 + \sum_{i=1}^{m} a_i S_i\right)$$

$$1 + \frac{1}{2} K_X |\Sigma| - \frac{1}{2}\Sigma^2 = \frac{(|a_0|-1)(|a_0|-2)}{2} - \sum_{i=1}^{m} \frac{|a_i|(|a_i|-1)}{2}$$

$$= \frac{|a_0|^2 - 3|a_0| + 2}{2} - \sum \frac{a_i^2 - |a_i|}{2}$$

$$= \frac{(a_0)^2}{2} - \sum \frac{a_i^2}{2} - \frac{3}{2}|a_0| + 1 + \sum_{i=1}^{m} \frac{|a_i|}{2}$$

$$= \frac{1}{2}\Sigma^2 + 1 + \frac{1}{2}\left(-3 |a_0| + \sum |a_i|\right)$$

$$= 1 + \frac{1}{2}\Sigma^2 + \frac{1}{2}\left(-3|a_0| + \sum_{i=1}^{m} |a_i|\right)$$

$$\therefore K_X = -3S_0 - \sum_{i=1}^{m} S_i$$

Thm Conj (KM)

$g(\Sigma) = g$. d.S. $S$ : gen $\in H_2(\mathbb{C}P^2)$

$$\Rightarrow g \geq \frac{1}{2}(d-1)(d-2)$$

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$K = -3S$. can. class

$$2g-2 \geq K \cdot \Sigma + \Sigma^2$$

Adj ineq: $X$: smooth 4-mfd $b^+ > 1$. $\Sigma \subset X$ surface with $\Sigma^2 \geq 0$

$\Sigma^2 \geq 0$

$K$: Seib. basic class.

$\Sigma$: non-tor sur.

If $g \geq 1 \Rightarrow 2g-2 \geq |K \cdot \Sigma| + \Sigma^2$

Conj (Hamilton) $X$ : $\pi_1 = e$. $A \hookrightarrow X$.

$A^2 = N < 0$

$$N \leq \frac{1}{4-k}\left(c(X) + k(2g_A - 1)\right)$$

const

$k < 4$

If Thm $g_A = 0$ $X = \#\mathbb{CP}^2 \# 9\overline{\mathbb{CP}^2}$

$$33 \leq \frac{1}{4-k}(c(X) - k)$$

$$(4-k)\,33 \leq c(X) - k$$

$$-32k + 132 \leq c(X)$$

$$1 \quad \frac{132 - c(X)}{32} \leq k \leq 4$$

$$4 = 132 - 32 \cdot 4 \leq c(X)$$

Finashin Mikhalkin. $A \hookrightarrow K3$

$0 \geq \forall n \geq -86 \Rightarrow \exists A \hookrightarrow K3$

sphere

$A^2 = n$