Generalized Property R について

§0 Intro

> Thm (Property R) If 0-surgery on a knot $K \subset S^3$ yields $S^1 \times S^2$, then K is the unknot.

by Gabai (Foliation)

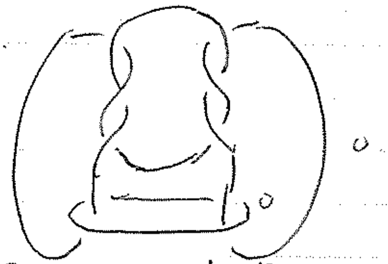
Goal : $\left\{ \begin{array}{l} \text{Generalized Property R の ~~[illegible]~~ 証明の outline.} \\ \text{Property } nR. \end{array} \right.$

特に反例候補の作り方.

§1. Generalized Property R

Fact $\underbrace{O^0 \quad \cdots \quad O^0}_{n} \approx \#^n S^1 \times S^2$

Example

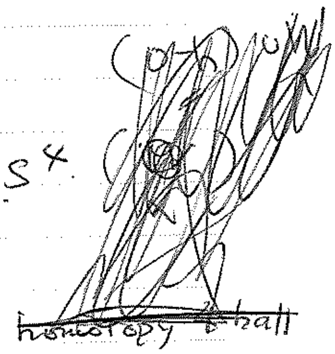


> **Conj.** (Generalized Property R)
> If integral-surgery along n-comp. link L yield $\#^n S^1 \times S^2$, then $L \xrightarrow[\text{slides}]{\exists} \underbrace{O^0 \cdots O^0}_{n\,J}$

§2. $\#^n S^1 \times S^2$ を生む link の性質

Lemma 1. If integral surgery along an n-comp. $L = L_1 \cup \cdots \cup L_n$ yield $\#^n S^1 \times S^2$

$$\begin{cases} \text{the framing of } L_i \ (i=1\cdots n) \text{ is zero.} \\ lk(L_i, L_j) = 0 \end{cases}$$

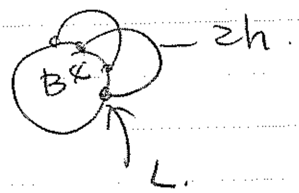


Lemma 2 (Hillman). Assume as above, then L gives rised to homotopy S^4.

In particular, $L \subset$ ∂(4 ball) bounds (disjoint) disks in homotopy 4-ball.

$\uparrow$ homotopy S^4 − (0-handle)

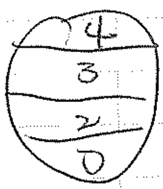
Proof. $X =$



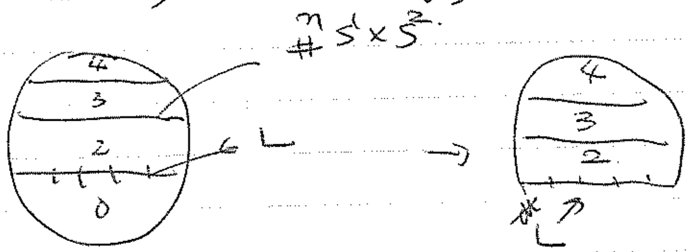
$$\partial X = \#^n S^1 \times S^2.$$

In particular, $X \underset{h.e.}{\simeq} \natural_n D^2 \times S^2$

$\therefore$ $X \cup$ (3 handles) = homotopy 4-ball.



homotopy 4 sphere



the cores of the 2-h's are the disks!

§3. Property 2R

A knot $K \subset S^3$ has property nR if K does not appear in any n-comp. counterexample of Generalized Property R

Conj. All knots have Property nR.

From now on $n=2$.

Prop The unknot has Property $2R$.

Thm If a fibered knot K does not have Property $2R$, then

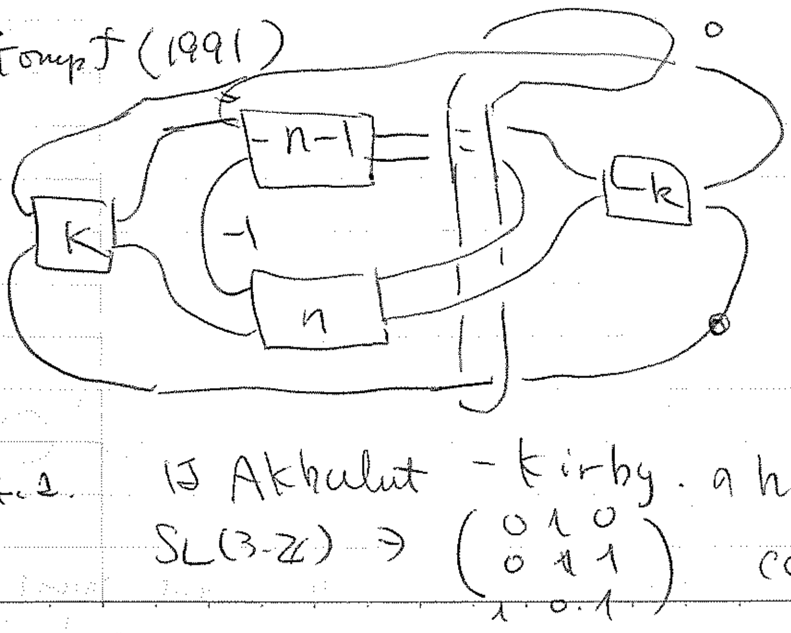
K' ~~does not~~ s.t. $\begin{cases} K' \text{ does not have Property } 2R. \\ g(K') < g(K) \end{cases}$

Cor. Let K be a fib. knot with $g(K)=1$
$\Rightarrow$ K has Property $2R$.

$T_{2,3} \# \overline{T_{2,3}}$ ← genus 2 slice fibered knot [5].

§4. A non-standard handle decomposition of S^4

Gompf (1991)



$\cup$ 4-h

(9) $\Sigma_{4.1}$. IJ Akbulut-Kirby. a homotopy 4-sphere

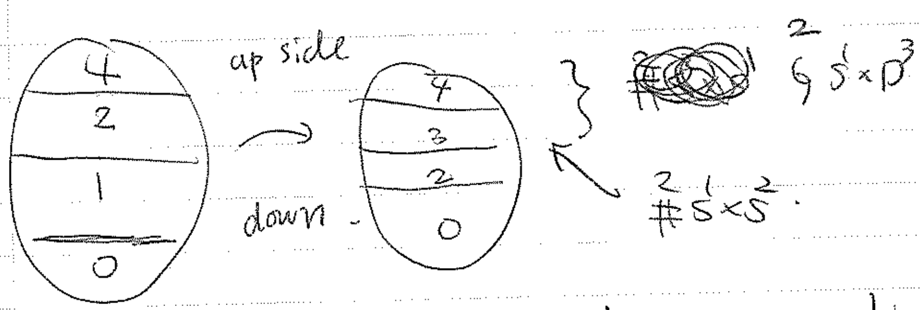
$SL(3,\mathbb{Z}) \ni \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ Cappell-Shaneson

$k=1$

$$P_n = \pi_1(\Sigma_{n,1}) = \left\langle x, y \;\middle|\; \begin{array}{l} xyx = yxy \\ y^{n+1} = x^n \end{array} \right\rangle$$

Note: P_n ($n \geq 3$) is potential counterexamples of Andrew Curtis conj.

§5. Property 2R potential count. ex.



$\Sigma_{n,k}$. $L_{n,k}$: 0-framed 2 comp link.

(2-handle attaching circles)

$k=1$, $n \geq 3$ case.

<u>Fact</u>. one comp of $L_{n,1}$ is $T_{2,3} \# \overline{T_{2,3}}$

another one is $T_{n,n+1} \# \overline{T_{n,n+1}}$

<u>Conj</u>. $L_{n,1}$ is counterexample of Generalized Property R.

If not counter exp.

, then $L_{n,1} \xrightarrow{\text{slide}} O^0 \; O^0$

$\Rightarrow$ $\Sigma_{n,1} \longrightarrow S^4$.

$\exists$ slide $\Rightarrow$ P_n is not count. example of A.C. conj.

~~Note: Bands~~

Note: band-sum of the 2-comp. of $L_{n,1}$

is slice. but possibly not ribbon.