Differential Topology '25
4-manifolds, skein lasagna modules, exotic structures, handle decompositions

Place: Ritsumeikan University(OIC campus)(room: H323) (hybrid)

2025/Mar/8-March/10

Abstract. The aim of this workshop is to study the skein lasagna module.

Program

3/8 (Sat)

14:00–15:00 Taketo Sano **Introduction to Khovanov Homology I: Construction**

Abstract. Khovanov homology is a link homology theory introduced by M. Khovanov in 2000, which categorifies the Jones polynomial. In this series of three talks, I will provide an introduction to the theory, aiming to provide the necessary background for the skein lasagna modules. In the first talk, I will focus on the construction of the Khovanov chain complex and its invariance. I will emphasize the underlying connections with Frobenius algebras and the 1+1-dimensional topological quantum field theory (TQFT) that form the foundation of the construction.

15:20–16:20 Taketo Sano **Introduction to Khovanov Homology II: Functoriality**

Abstract. Khovanov homology is functorial with respect to link cobordisms; namely, a surface between two links induces a homomorphism between the corresponding homology groups. In this second talk, I will explain how the cobordism map is constructed and how its invariance under isotopies is proven.

16:40–17:40 Nobuo Iida **On the Khovanov-Rozansky homology**

Abstract. Khovanov homology is a doubly graded vector space associated with a knot, which can be regarded as a categorification of the Jones polynomial in the sense that taking its graded Euler characteristic recovers the Jones polynomial. According to the Reshetikhin-Turaev theory, the Jones polynomial is a knot polynomial associated with the vector representation of the quantum group of $\mathfrak{sl}_2$, and polynomial invariants for $\mathfrak{sl}_n$ are also defined in a similar manner. Various constructions of knot homology that categorify these invariants are known, including the one by Khovanov and Rozansky. In this talk, we provide an overview of these constructions.

3/9 (Sun)

10:00–11:00 Hiroaki Karuo **Linear skein theory 1**

Abstract. The question of whether the structure of a 4-dimensional manifold is preserved when a 2-handle is attached reduces to the problem of whether Kirby moves can be performed in the attaching region of the 2-handle, which is an annular neighborhood of a knot in the 3-sphere. A similar perspective arises in the skein lasagna module, where it is necessary to assign an appropriate color to the knot. In this talk, we will first review how colors were assigned in the construction of quantum invariants of 3-manifolds and then explain what should ideally hold in the case of 4-dimensional manifolds. I will be responsible for this first part of the talk.

11:20–12:20 Wataru Yuasa **Linear skein theory 2**

Abstract. I will be responsible for the latter part of the abstract of Linear Skein Theory I. In particular, I will focus on the construction of the Kirby color in the annular Bar-Natan category using the dotted Temperley-Lieb category introduced by Hogancamp, Rose, and Wedrich.

Lunch

14:00–15:00 Masaki Ogawa **Constrution of Skein lasagna module**

Abstract. We introduce a cobordism called Lasagna filling and an associated algebraic structure called the Lasagna algebra. Using the functoriality of Khovanov-Rozansky homology, we first show that Khovanov-Rozansky homology carries the structure of a Lasagna algebra. We then introduce the notion of Lasagna modules. Understanding this construction reveals that Lasagna modules provide a framework not only within the setting of Khovanov-Rozansky homology but also for various homology theories of knots that satisfy functoriality properties.

15:20–16:20 Masaki Ogawa **Skein lasagna modules for 2-handlebodies**

Abstract. In this talk, we discuss the result of Manolescu and Neithalath, which considers the Lasagna module introduced in the previous talk in the context of 2-handlebodies. The computation of the Lasagna module for a 2-handlebody reduces to computing the Khovanov-Rozansky homology of a cable of the attaching link of the 2-handle, modulo a certain equivalence relation. This is referred to as cabled Khovanov-Rozansky homology. By examining the proof of this result, we gain insight into the concrete construction of elements of the Lasagna module and their correspondence with elements of the cabled Khovanov-Rozansky homology.

16:40–17:40 **Skein lasagna module & handle decompositions**

Abstract. I will give an overview of the paper "Skein Lasagna Modules and Handle Decompositions" by Manolescu, Walker, and Wedrich. This paper provides a general formula for describing skein lasagna modules based on the handle decomposition of a 4-dimensional manifold. In this talk, I will focus in particular on the treatment of 1-handles.

18:30– Banquet

3/10 (Mon)

10:00–11:00 Taketo Sano **Introduction to Khovanov Homology III: Lee Homology and the Rasmussen Invariant**

Abstract. By modifying the defining Frobenius algebra of Khovanov homology, we obtain deformed versions of the theory. One such deformation is Lee homology, which gives rise to an integer-valued knot invariant, known as the Rasmussen invariant. In this third talk, I will introduce these concepts and discuss their notable applications, including Rasmussen ' s combinatorial reproof of the Milnor conjecture and Piccirillo ' s proof that the Conway knot is not slice.

11:20–12:20 Masaki Ogawa **Detecting exotic 4-manifolds via skein lasagna modules**

Abstract. In this talk, I will discuss the results of Ren and Wills on detecting exotic differential structures using skein lasagna modules. In particular, after introducing the lasagna s-invariant used in the proof, I will give an overview of the actual computations performed using this invariant.

Informal seminar

14:00–16:00 Taketo Sano **Cobordism Interpretation and Computation of Invariants**

Abstract. Rasmussen's s-invariant is an integer-valued knot invariant derived from Khovanov homology, with remarkable applications in low-dimensional topology, such as providing a combinatorial proof of the Milnor conjecture. The s-invariant is defined using a filtration on the homology groups induced by quantum grading, but extracting its geometric meaning is not straightforward. In this talk, we provide a cobordism-theoretic interpretation of the s-invariant, based on Bar-Natan ' s formulation of Khovanov homology using tangles and cobordisms. This interpretation enables the computation of the s-invariant from a tangle decomposition of the knot. As an application, we show how this approach determines the s-invariant for an infinite family of certain pretzel knots.