Lens space surgery and a classification.

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§1. Dehn surgery.

M : 3-mfd. ($\mathbb{Z}HS^3$)

$K \subset M$ a knot

$$M_{p/q}(K) = M - N(K) \cup_{\varphi} S^1 \times D^2$$

$$\varphi : \partial D^2 \longleftrightarrow \underset{\uparrow \text{ meridian.}}{p\,m} + \underset{\uparrow \text{ longitude}}{q\,l}$$

Q. X : mfd.
when X is homeo to $M_{p/q}(K)$?

Lens space surgery. $M = S^3$, $X = L(p,q)$

↑ Main problem here.

or M : L-space $\mathbb{Z}HS^3$.

we focus on $q=1$ surgery
integral surgery

§2. Lens space surgery invariant.

M : $\mathbb{Z}HS^3$.

Supp. $M_p(K) = L(p, p_2) \supset \tilde{K}$ dual knot

$[\tilde{K}] \in H_1(L(p, p_2)) \cong \mathbb{Z}/p\mathbb{Z}$

↑ take of a core of $g=1$ Heeg. split of $L(p, p_2)$

$[\tilde{K}] = k \in \mathbb{Z}/p\mathbb{Z}$

(p, k) is called lens surg. parameter.

§3. Berge's knot.

M : $\mathbb{Z}HS^3$

$K \subset M$ is Berger's knot (or double primitive knot)

1) $M = H_0 \cup_{\Sigma_2} H_1$ $g=2$ Heeg. split.
2) $K \subset \Sigma_2$
3) $H_i(K)$:(2-handle attach. along K) are both solid torus.

$B_M = \{K : \text{Berge's knot in } M\}$

Prop 1. (B) $\forall K \in B_M$ $\exists p$.

$M_p(K) = L(p, p_2)$

Prop 2. (B) $\forall (p, k)$ relatively prime. $\exists (M, K)$ $K \in B_M$.
from $\exists p$ $M_p(K) = L(p, p_2)$

List 1. (B) The below is a member of B_{S^3}
(1)
$\vdots$ excep (6).
(10)

These are (pb) expression, (Each has infinite)
$\{(1), \cdots (10)\} \subset B_{S^3}$

Prop 3. (B) List 1 $\subset B_{S^3}$

Thm (Greene). $B_{S^3} \subset$ List 1

Conj B_{S^3} are all knots yielding Lens sp. in S^3.

§4. My research.
$\tilde{\mathcal{M}} = \{$L-space $ZHS^3\}$

$$B^0_{\tilde{\mathcal{M}}} := \underbrace{B_{\tilde{\mathcal{M}}}}_{\displaystyle \overset{\shortparallel}{\bigsqcup_{M \in \tilde{\mathcal{M}}}} B_M} \setminus B_{S^3}$$

List 2. (T).

(A) ← (p.k) expressions
⋮
(K) ←

(A) ~ (J) have infinite. (p.k)
(K) has single pair (191, 15)
$= (191, \overline{51})$

Prop 4 (T) List 2 $\subset B_{\overline{\Sigma}(2,3,5)} \subset B_{\mathring{M}}$

Conj $B_{\mathring{M}} \subset$ List 2.

Main Thm. (T.) This Conj is almost all true.

§5. Outline of proof
— in order to prove —

M : L-space $\subset S^3$
$M_p(K) = L(p, p_2) \Longrightarrow (p.k) \in$ List 2.
(p.k)

§6 Contin^u^ed faction.

$$
\begin{aligned}
k^2 &= \tau_0 p - p_2 \\
p &= \tau_1 p_2 - p_3 \\
&\vdots \\
p_{n-1} &= \tau_{n-1} p_n - p_{n+1} \\
p_n &= \tau_n p_{n+1}
\end{aligned}
$$

least abs. remainder

$\left(|p_{i+1}| \le \left\lfloor \frac{|p_i|}{2} \right\rfloor \right)$

$p_{n+1} = \pm 1$

Lemma 1 If $M_p(K) = L(p, p_2)$
(p, k) then.
there. ex. $1 \le u < v \le n+1$

$$
k = \begin{cases}
|p_u| & \cdots (☆) \\
2|p_u| & \cdots (☆☆) \\
\left\{ \begin{matrix} |p_u| \\ |p_{u-1}| - |p_u| \end{matrix} \right. + \left\{ \begin{matrix} |p_v| \\ |p_{v-1}| - |p_v| \end{matrix} \right. & \\
|p_{u-1}| - \left\{ \begin{matrix} |p_v| \\ |p_{v-1}| - |p_v| \end{matrix} \right. &
\end{cases}
$$

proof. Use flatness of Δ_K

$\uparrow$

$\forall$ coeff a_i of Δ_K

$|a_i| \le 1$

rem. flatness is neces. cond. of Ozsvath Szabo's L-space surgery.

§ 7. classification.

<u>Lemma 2</u>

(☆) $\Longleftrightarrow$ Berge's (1)

(☆☆) $\Longleftrightarrow$ Berge's (2)

$u=1$ $\Longleftrightarrow$ Berge's (7), (8)

$u=2$ $\Longleftrightarrow$ Berge's (3), (4), (9), (10)
my (A), (C), (D), (E)
(F), (G), (H).

$u \geq 3$ $\Longleftrightarrow$ Berge's (5)
& $k = |p_u| \pm |p_d|$ my (I) (J), (B)

<u>proof</u> alternating of Δ_K.

↑

nonzero coff. of Δ_K.
is alternating sign

Rem2. alternating cond is also nerc. cond
to yield L-space by
Dehn surgery.

Remaining case.
$u \geq 3$ and k is other case.

Cor.(Criterion)

Cor (Criterion)

(p, k) is a para of $B_{\tilde{\mu}}$

$\Leftrightarrow$ $\Delta_{p,k} :=$ the remainder of

$$\frac{(t^{kl}-1)(t-1)}{(t^k-1)(t^l-1)}$$ when dividing it by $t^p - 1$

$\Delta_{p,k}$ is flat & alternating.

, where. $(k, l) = 1$

gcd

$k \cdot l = 1 \quad (p)$