Algebraic independence of the values and the derivatives of a certain family of Lambert-type series

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Let $\alpha_1, \ldots, \alpha_r$ be algebraic numbers with $0 < |\alpha_i| < 1$ $(1 \leq i \leq r)$ and $\{R_k\}_{k \geq 0}$ a certain linear recurrence including Fibonacci sequence. We consider the Lambert-type series $H_i(x) = \sum_{k=0}^{\infty} \alpha_i^{R_k}/(1 - \alpha_i^{R_k} x)$ $(1 \leq i \leq r)$, where x is a complex variable. For each i $(1 \leq i \leq r)$, Tanaka proved the algebraic independence of the infinite set $\mathcal{H}_i$ consisting of the values and the derivatives of any order of $H_i(x)$ at any distinct algebraic numbers except its poles. Moreover, the speaker and Tanaka showed that, if $\alpha_1, \ldots, \alpha_r$ are multiplicatively independent, then the infinite set $\mathcal{H} = \cup_{i=1}^{r} \mathcal{H}_i$ consisting of the values and the derivatives of any order of $H_1(x), \ldots, H_r(x)$ at any distinct algebraic numbers except their poles is algebraically independent. Extending these results, we present in this talk a necessary and sufficient condition on $\alpha_1, \ldots, \alpha_r$ for the infinite set $\mathcal{H}$ to be algebraically independent.