Workshop "Number Theory and Ergodic Theory"
(February 10 – 12, 2024 at Kanazawa University Satellite Plaza)

Abstracts of talks

Hiroshi Fujisaki (Kanazawa University)

Title: On topological entropies of the subshifts associated with the stream version of asymmetric binary systems

Abstract: The stream version of asymmetric binary systems (ABS) invented by Duda is an entropy coder for information sources with a finite alphabet. It has the state parameter l of a nonnegative integer and the probability parameter p with $0 < p < 1$. First we observe that the edge shift X_G associated with the stream version of ABS has the topological entropy $h(X_G) = \log 2$. Then we define the edge shift X_H associated with output blocks from the stream version of ABS, and show that $h(X_H) = h(X_G)$. The encoding and decoding algorithms for the stream version of ABS establish a bijection between X_G and X_H. We consider the case where $p = 1/\beta$ with the golden mean $\beta = (1 + \sqrt{5})/2$, and show that X_G and X_H are conjugate for $l = 7$.

Keywords: topological entropy, shift space, edge shift, conjugacy, asymmetric binary systems(ABS)

Hajime Kaneko (University of Tsukuba)

Title: Analogue of Markoff and Lagrange spectra on one-sided shift spaces

Abstract: Recently, multiplicative analogue of Markoff-Lagrange spectrum was investigated, which is related to the fractional parts of geometric progressions. This analogy is investigated via limit sup words on shift spaces. In particular, Dubickas showed that the minimal limit points of certain spectra are denoted in terms of a substitution. In this talk, we consider general ordered shift spaces and analogue of Markoff and Lagrange spectra. In our main results, we determine the minimal limit points and discrete parts. In particular, we show that the minimal limit points are denoted in terms of S-adic sequences, where S is an infinite set of substitutions. This is a joint work with Wolfgang Steiner.

Keywords: Markoff and Lagrange spectra, discrete part, S-adic sequences

Yasushi Nagai (Shinshu University)

Title: Overlap algorithm for general S-adic tilings

Abstract: We investigate the question of when a tiling has pure point spectrum, for the class of S-adic tilings, which includes all self-affine tilings. The overlap algorithm by Solomyak is a powerful tool to study this problem for the class of self-affine tilings. We generalize this algorithm for general S-adic tilings, and apply it to a class of block S-adic tilings to show almost all of them have pure point spectra. This is a joint work with Jorg Thuswaldner.

Keywords: tiling, pure point spectrum, overlap algorithm

Hideki Tsuiki (Kyoto University)

Title: TriMata and Tiling

Abstract: 3D TriMata is an assembly kit composed of pieces shaped as two connected hexahedra, derived by dividing a regular regular tetrahedron into four identical shapes. This kit enables the construction of models based on the Sierpinski tetrahedron. Based on this fact, we show two 3D tiles: one allows for periodical tiling, but when both tiles are required to be used, then only aperiodic tiling is possible.

Keywords: aperiodic tiling, Sierpinski tetrahedron

Tadahisa Hamada (University of Tsukuba)

Title: Transition process of factor graphs of Sturmian words

Abstract: This study elucidates the properties of Sturmian words by using the graph repre-

sentation of factors, known as factor graph, which was introduced by Rauzy (1982).
Keywords: Sturmian word, factor graph

Teturo Kamae (Osaka Metropolitan University)
Title: Infinite self-shuffling words
Abstract: An infinite word $x = x_0x_1x_2\cdots$ over an alphabet A is called self-shuffling if there exists a partition $N_1 \cup N_2 = \{0, 1, 2, \cdots\}$ such that both x restricted to N_1 and N_2 are x. An easy example is Fibonacci word x which is invariant under the substitution $\phi : 0 \to 01, 1 \to 0$, since in this case we have $x = \phi^2(x) = \phi(x_0)x_0\phi(x_1)x_1\cdots$, and hence $\phi(x_0)\phi(x_1)\cdots = x_0x_1\cdots = x$. We prove a necessary and sufficient condition to be self-shuffling and prove that the Thue-Morse word and any Sturmian word with non-zero intercept are self-shuffling. This is a joint work with Emilie Charlier, Svetlana Puzynina and Luca Zamboni.
Keywords: self-shuffling words, Thue-Morse word, Sturmian words

Yohei Tachiya (Hirosaki University)
Title: Linear independence of infinite series related to the Thue-Morse sequence along powers
Abstract: The Thue-Morse sequence along the powers has been studied by several authors. In this talk, we give a linear independence result for the values of infinite series generated by these sequences. This generalizes an irrationality result of E. Miyanohora(2023) who treated the case of the squares. This is a joint work with Michael Coons.
Keywords: Thue-Morse sequence, Linear independence, Pisot numbers, Salem numbers

Masamichi Yoshida (Osaka Metropolitan University)
Title: Signed Expansion and associated Odometer based on Cubic Pisot Unit (Joint work with Fumichika Takamizo)
Abstract: First signed expansion of integers based on a cubic Pisot unit is introduced. Then we study the associated odometer and domain exchange.
Keywords: Signed expansion, odometer, domain exchange

Nathaniel Nollen (University of the Philippines-Diliman)
Title: Number of Nonzero Digits in Quaternion Expansions
Abstract: Let $\{\eta_1, \eta_2, \eta_3, \eta_4\}$ be an $\mathbb{R}$-basis of the skew-field $\mathbb{H}$ of real quaternions. We consider the lattice $\mathcal{L} := \mathbb{Z}\eta_1 + \mathbb{Z}\eta_2 + \mathbb{Z}\eta_3 + \mathbb{Z}\eta_4$ and the fundamental domain

$$\chi := \{t_1\eta_1 + t_2\eta_2 + t_3\eta_3 + t_4\eta_4 : t_1, t_2, t_3, t_4 \in [0, 1)\}.$$

For $q \in \mathbb{H}$ with $|q| > 1$, define the transformation T_q on χ by $T_q(z) = qz - d(z)$ where $d(z)$ is the unique element d of $\mathcal{L}$ such that $qz - d \in \chi$.

The quaternion expansion of $z \in \chi$ in base q is the sum

$$z = q^{-1}d_1 + q^{-2}d_2 + q^{-3}d_3 + \cdots,$$

where $d_j = d(T^{j-1}(z))$. We identify this expansion with the sequence $(d_1, d_2, d_3, \dots)$ of digits d_i's. In this presentation, we provide some results on the number of nonzero digits in quaternion expansions as a measure of digit complexity.
Keywords: quaternion, expansions, fundamental domain, lattice, skew field

Asaki Saito (Future University Hakodate)
Title: Exact binary expansion of real algebraic integers using Newton's method
Abstract: We introduce a method for computing exact binary expansions of real algebraic integers using Newton's method. This method allows us to speed up the true orbit pseudorandom number generation that we have proposed so far. This is a joint work with Akihiro Yamaguchi of Fukuoka Institute of Technology.
Keywords: Newton's method, algebraic integer, binary expansion, pseudorandom number

Mako Toyosumi (Japan Women's University)

Title: Some complex continued fractions with coefficients in a submodule

Abstract: We discuss a complex analogue of continued fractions of even integers in the real case. This type of complex continued fractions is first considered by J. Hurwitz for the Gaussian field. In this talk, we consider the case of the Eisenstein field. We also consider the analogue of the odd integer case.

Keywords: complex continued fraction

Dong Han Kim (Dongguk University - Seoul)

Title: Continued fraction algorithm for rational numbers of certain parity types

Abstract: There are three different kinds of rational numbers according to the parity of the numerator and denominator, say odd/odd, even/odd and odd/even rational numbers. For a given irrational number, we consider the properties of best rational approximations of given parities. We study continued fraction algorithms to find best rational approximations by rational numbers of given parities and compare the algorithms with continued fraction expansions. This is joint work with Seul Bee Lee and Lingmin Liao.

Keywords: Continued fraction, intermediate convergents, Diophantine approximation

Shin-ichi Yasutomi (Toho University)

Title: Simultaneous convergent continued fraction algorithm with two places and the potential of ergodic properity.

Abstract: Let p be a prime number and K be a field with embeddings into $\mathbb{R}$ and $\mathbb{Q}_p$. We propose an algorithm that generates continued fraction expansions converging in $\mathbb{Q}_p$ and is expected to simultaneously converge in both $\mathbb{R}$ and $\mathbb{Q}_p$. This algorithm produces finite continued fraction expansions for rational numbers. In the case of $p = 2$ and if K is a quadratic field, the continued fraction expansions generated by this algorithm converge in $\mathbb{R}$, and they are eventually periodic or finite. For an element α in K, let p_n/q_n denote the n-th convergent. There exist constants u_1 and u_2 in $\mathbb{R}_{>0}$ with $u_1 + u_2 = 2$, and constants C_1 and C_2 in $\mathbb{R}_{>0}$ such that $|\alpha - p_n/q_n| < C_1/|q_n|^{u_1}$ and $|\alpha - p_n/q_n|_2 < C_2/|q_n|^{u_2}$. Here, $|\cdot|_2$ represents the 2-adic distance. For prime numbers $p > 2$, we present numerical experiences. Furthermore, this algorithm has the potential for natural extension, with mention of its ergodic features being captured.

Keywords: continued fraction, simultaneous approximation

Haruyoshi Tanaka (Naruto University of Education)

Title: Higher-order asymptotic behaviours of pressure functionals and statistical representations of the coefficients

Abstract: We study the higher-order asymptotic behaviour of the pressure functional $\epsilon \mapsto P(\varphi+\epsilon\psi)$ with potentials φ and ψ on a countable Markov shift X. We show that if the transition matrix of X is finitely primitive, the potentials φ and ψ are real-valued locally Hölder continuous functions on X, and a sufficient condition for an asymptotic expansion of $\epsilon \mapsto P(\varphi + \epsilon\psi)$ is satisfied, then the 3-th coefficient of this expansion has a limit representation which looks like the asymptotic variance well. In this study, we also give an explicit formula for the behaviour of $\mu((\sum_{i=0}^{m-1}(\psi - \int_X \psi\, d\mu) \circ \sigma^i)^k)$ as $m \to \infty$ with the Gibbs measure μ for φ and the shift transformation σ. In application, we apply the convergence rate of the mean ergodic theorem in $L^k(\mu)$ with even integer k to our result.

Keywords: asymptotic variance, topological pressure, countable Markov shift

Soonki Hong (Postech)

Title: Weak Ramanujan property of standard non-uniform arithmetic quotient for PGL_3 and PGL_4

Abstract: As a generalization of Morgenstern's definition of the Ramanujan diagram, Samuels defined Ramanujan complexes on the quotient of the Bruhat-Tits building B_d associated to $PGL_d(\mathbb{F}_q((t^{-1})))$ by a congruence subgroup of $PGL_d(\mathbb{F}_q[t])$. She proved that if d is prime, the

quotient is not Ramanujan but is weak Ramanujan. In this talk, we will prove that the quotient $PGL_d(\mathbb{F}_q[t])\backslash B_d$ is weak Ramanujan when $d = 3, 4$. This is joint work with S. Kwon.
Keywords: Ramanujan complex, Bruhat-Tits building, spectrum of adjacency operators

Kota Saito (University of Tsukuba)

Title: The simple normality of the square root of two and the Riemann zeta function

Abstract: A real number x is called a simply normal number to base b if the base-b expansion of x has each digit $0, 1, \ldots, b-1$ appearing with average frequency tending to $1/b$. We do not know whether π, e, $\log 2$, $\sqrt{2}$, and so on are simply normal or not. In this talk, we report discovering a relation between the frequency that the digit 1 appears in the binary expansion of $\sqrt{2}$ and a mean value of the Riemann zeta function on arithmetic progressions. As a consequence, we show that

$$\lim_{l\to\infty} \frac{1}{l} \sum_{0<|n|\leq 2^l} \zeta\left(\frac{2n\pi i}{\log 2}\right) \frac{(-1)^n}{n} = 0$$

if and only if $\sqrt{2}$ is simply normal to base 2. This is joint work with Yuya Kanado (Nagoya University).
Keywords: simple normality, Riemann zeta function, Perron's formula, functional equation