

代数IB宿題9

2015-6-12

1. Let $\lambda \in \mathbb{C}$, and let M_λ be the Verma module of highest weight λ , i.e., the infinite-dimensional representation freely generated by a highest weight vector v_λ . We can present M_λ as a direct sum of one-dimensional H -eigenspaces $\bigoplus_{i=0}^{\infty} \mathbb{C}_{\lambda-2i}$, with F producing isomorphisms $\mathbb{C}_{\lambda-2i} \rightarrow \mathbb{C}_{\lambda-2i-2}$ for all $i \geq 0$. Show that M_λ is irreducible if and only if $\lambda \notin \mathbb{Z}_{\geq 0}$. Furthermore, show that if $\lambda \in \mathbb{Z}_{\geq 0}$, then $M_{-\lambda-2}$ is a submodule, and the quotient $M_\lambda/M_{-\lambda-2}$ is irreducible of highest weight λ .

提出期限: 6月19日授業中