18.784 Homework Set 6
Due Friday, March 12, 2010.

Part I (PE-CK, 3/8/10)

1. Draw fundamental domains for the congruence groups $\Gamma_0(5)$ and $\Gamma_1(3)$.

2. Show that for integers $k \geq 3$ $N \geq 1$, and vector $\vec{a} = (a_1, a_2) \in (\mathbb{Z}/N\mathbb{Z})^2$, the Eisenstein series

$$G_k^{\vec{a}} = \sum_{\substack{\vec{m} \in \mathbb{Z}^2 \\ \vec{m} \equiv \vec{a}(N)}}{}^{*} \frac{1}{(m_1 z + m_2)^k}$$

is a modular form of weight k for $\Gamma(N)$. Here, the star in $\sum^*$ means we omit the summand $m = (0, 0)$.

Part II (AH-QY, 3/10/10)

1. Find the order of $PSL_2(\mathbb{Z}/N\mathbb{Z})$ when N is prime.

2. Show that every curve of the form $y^2 = (x-a)(x-b)(x-c)$ can be written in Weierstrass form: $y^2 = 4x^3 - g_2 x - g_3$ after a change of coordinate variables. Using the fact that the modular function j is surjective, show that all such curves are isomorphic to $\mathbb{C}/\Lambda$ for some lattice Λ.