18.086 Problem Set 4 due Friday, April 17, 2009.

Please email your code to the grader, and turn in your answers in class.

Movie Villain's stealth yacht has malfunctioned in open water at coordinates $(20, 10)$, and is stationary (coordinates are in kilometers). Movie Hero has just discovered the location, and has activated a robot sailboat with a payload of explosives. The sailboat is stationary at $(0, 0)$ when it is activated at time $t = 0$. We have the following environmental conditions:

(1) There is no land nearby (i.e., we may assume the sailboat can travel anywhere in $\mathbb{R}^2$).
(2) The water is stationary (i.e., there is no current).
(3) There is a steady wind blowing in the negative y direction.

Movie Hero's sailboat has the following ideal properties:

(1) The sailboat has a monster keel, that ensures that any force from the wind only results in acceleration parallel to the direction the boat is pointing (you should be thinking of an orthogonal projection).
(2) There is no hydrodynamic drag impeding movement in the direction the boat is pointed.
(3) There is no aerodynamic drag on the sailboat.
(4) The sail produces a pure lift force proportional in magnitude to the apparent wind speed ($= \|v_{wind} - v_{boat}\|$), and in a perpendicular direction to the apparent wind. Since there are two choices of perpendicular direction, we will assume Movie Hero chooses the one with a forward component (if it exists).
(5) The sailboat has a great big rudder, and is able to change direction in negligible time, while maintaining speed.

You have the following numerical data:

(1) The mass of the sailboat is 10^4 kg.
(2) The wind is moving at 10 m/s.
(3) The conversion from apparent wind speed to force on the sail is given by the constant 10^3 kg /s.

Your mission is to help Movie Hero by writing a program to find the minimal-time trajectory for her boat to pass from $(0, 0)$ to $(20, 10)$. Plot the optimal trajectory, and write down the elapsed time and final speed.

Optional:

If you think this is too easy, try removing some of the ideal assumptions regarding the sailboat. In particular, sailboats in the real world experience hydrodynamic and aerodynamic drag roughly proportional to the square of the speed relative to the water (resp. air), and boats can be blown sideways. Make sure your drag parameters aren't too large, or the boat won't be able to reach Movie Villain's yacht. If you try this, make sure you document the changes to your code.

Even more optional:

Suppose Movie Villain's boat is at $(0, 10)$. What happens to the uniqueness of the optimal solution? Suppose we replace the keel and rudder with a hydrofoil that is only able to exert a certain amount of lateral force against the water for turning purposes. What kind of trajectory is optimal?