18.086 Problem Set 3 due Wednesday, April 1, 2009.

Please email your code to the grader, and turn in your answers in class.

(1) Write a program to solve the inviscid Burgers equation $u_t + uu_x = 0$ by a finite-volume method. Given the initial conditions

$$U(x,0) = \begin{cases} 0 & x < -2 \\ 1 & -2 \leq x < 0 \\ 1-x & 0 \leq x < 1 \\ 0 & x \geq 1 \end{cases}$$

plot $U(x,2)$ and $U(x,5)$, and $U(x,7)$.

(2) Consider the following 2D Poisson problem:

- The domain is an L-tromino, i.e., a union of three unit squares, with centers $(1/2,1/2)$, $(1/2,3/2)$ and $(3/2,1/2)$.
- $-\Delta u = 1$ in the interior of the domain.
- $u = f$ on the vertical boundaries, i.e., on the line segments from $(0,0)$ to $(0,2)$, from $(1,1)$ to $(1,2)$, and from $(2,0)$ to $(2,1)$.
- $\frac{\partial u}{\partial n} = \frac{\partial f}{\partial n}$ on the horizontal boundaries, i.e., on the line segments from $(0,2)$ to $(1,2)$, from $(1,1)$ to $(2,1)$, and from $(0,0)$ to $(2,0)$.
- $f = xy^3 - x^2$.

Use the following methods on a "decent" size grid, i.e., one that gives reasonable precision but doesn't crush your computer:

(a) Elimination with reordering
(b) Gauss-Seidel iteration
(c) A multigrid method
(d) Conjugate gradients

Be sure to run the iterative methods until the accuracy is on the order of the discretization error. Compare the runtimes using, e.g., `tic` and `toc`.