James-Stein Estimator of Spiked Leading Eigenvector of High-dimensional Covariance Matrix

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Recently, a James-Stein shrinkage (JS) estimator has gained attention as a powerful tool for estimating the leading eigenvector of covariance matrices. In a series of seminal works, the efficacy of the JS estimator has been demonstrated under a spiked covariance model, using the high-dimensional, low-sample-size (HDLSS) asymptotic regime, where the number of variables increases while the sample size n remains fixed. We extend the application of the JS shrinkage to the regime of $n, p \to \infty$ with appropriate rate and reveal a key condition involving a signal-to-noise ratio, for the JS estimator to be useful. This approach utilizes geometric representation, a phenomenon that arises in high-dimensional asymptotics, to interpret the structure of parameters and estimators on a sphere within a lower-dimensional space. Furthermore, we develop shrinkage estimators for principal component variance and scores, enabling their application in high-dimensional principal component analysis.