TALK SUMMARY

1. The Riemann Zeta-Function and Hecke Congruence Subgroups. II
2. Three problems of Atle Selberg (1917–2007)

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In the first talk I shall present a succinct account of my recent work which has been uploaded to arXiv with the id: `0709.2590` [math.NT] under the same title as above. That is in fact a rework of my old file on an explicit spectral decomposition of the mean value

$$M_2(g;A) = \int_{-\infty}^{\infty} \left|\zeta\left(\tfrac12+it\right)\right|^4 \left|A\left(\tfrac12+it\right)\right|^2 g(t)dt,$$

which has been left unpublished since September 1994, though its summary account is given in [1] (see also [2, Section 4.6]); here

$$A(s) = \sum_n \alpha_n n^{-s}$$

is a finite Dirichlet series and g is assumed to be even, regular, real-valued on $\mathbb{R}$, and of fast decay on a sufficiently wide horizontal strip. At this occasion I have added greater details including a rigorous treatment of generalized Kloosterman sums associated with arbitrary $\varGamma_0(q)$ as well as an in-depth treatment of the Mellin transform

$$Z_2(s;A) = \int_{1}^{\infty} \left|\zeta\left(\tfrac12+it\right)\right|^4 \left|A\left(\tfrac12+it\right)\right|^2 t^{-s}dt$$

which was scantly touched on in [1]. My result on $Z_2(s;A)$ seems to shed light on the nature of the plain sixth power moment

$$M_3(g;1) = \int_{-\infty}^{\infty} \left|\zeta\left(\tfrac12+it\right)\right|^6 g(t)dt.$$

I shall set out certain ensuing problems which are concerned with the distribution of eigenvalues of the hyperbolic Laplacian acting over $L^2(\varGamma_0(q)\backslash\mathrm{PSL}(2,\mathbb{R}))$ with varying Stufen.

In the second talk I shall speak about the three problems which were personally shown to me by Atle Selberg who passed away on 6 August 2007. With this, I shall try to briefly relate three of his many great contributions – the elementary proof of the prime number theorem, the Λ^2 sieve, and the theory of the zeta-functions.

References

[1] Y. Motohashi. The Riemann zeta-function and Hecke congruence subgroups. RIMS Kyoto Univ. Kokyuroku, **958** (1996), 166–177.

[2] —. *Spectral Theory of the Riemann Zeta-Function*. Cambridge University Press, Cambridge 1997.