Coding of irrational rotation: recursively renewable structure.

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We study words generated by coding of rotation $x \mapsto x + \xi$ on the torus $\mathbb{R}/\mathbb{Z}$ with an irrational $\xi \in [0, 1)$. The well known example, Sturmian word, is given by 2-decomposition $[0, 1) = [0, 1 - \xi) \cup [1 - \xi, 1)$ coded by two letters 0 and 1. In this talk, general k-decomposition

$$[0, \omega_1) \cup [\omega_1, \omega_2) \cup \cdots \cup [\omega_{k-1}, 1)$$

using k letters $\{0, 1, \ldots, k - 1\}$ is investigated. One can show that such word is *recursive $(k+1)$-renewable*, that is, described as an element of an inverse limit directed by substitutions on $k + 1$ letters. The substitutions sensitively depend on the choice of parameters. Then we discuss when such process is stationary, i.e., given by eventually periodic recursion. We can show that this happens when and only when all parameters: the initial point, the angle ξ, and the division points ω_i, are simultaneously in a fixed quadratic field. (A joint work with Masayuki Shirasaka.)