SUMS OF FIVE CUBES OF PRIMES IN SHORT INTERVALS

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One of the famous and still unsettled problems in additive prime number theory is the conjecture that every sufficiently large integer, satisfying some natural congruence conditions, can be written as the sum of four cubes of primes. Although the present methods lack the power to prove such a strong result, Hua [*On the representations of a number as the sum of two cubes*, Math. Z. 44 (1938), 335–346] has been able to show that every sufficiently large odd integer is the sum of nine cubes of primes. He also established that almost all integers $n \in \mathfrak{N} = \{n \in \mathbb{N} : n \equiv 1 \,(\mathrm{mod}\ 2),\ n \not\equiv 0, \pm 2 \,(\mathrm{mod}\ 9),\ n \not\equiv 0 \,(\mathrm{mod}\ 7)\}$ can be expressed as the sum of five cubes of primes. Afterwards, the work of Hua has been continued by several authors.

We gain further insight into the problem of representing integers as the sum of five cubes of primes by averaging over short intervals only. Given arbitrary $A, \varepsilon > 0$ and a sufficiently large x, we prove that the number of integers $n \in \mathfrak{N} \cap (x, x+H]$, which cannot be represented as the sum of five cubes of primes is at most $\mathcal{O}(H(\log x)^{-A})$, provided that $x^{2/3+\varepsilon} \le H \le x$.