Hankel determinants and substitutions – some results and problems

Jun-ichi Tamura

We consider determinants $h_n = h_n[w] := \det(w_{i+j-1})$ of size n coming from an infinite word $w = w_1w_2w_3...$ $(w_i \in A)$ over an infinite, or a finite alphabet A such that w is a fixed point of a substitution, where h_n is considered to be an element of $\mathbf{Z}[A]$ with independent variables $a \in A$.

First, we give a short survey for the results on h_n when w is

(1) the Thue-Morse word,

(2) a word of Fibonacci type

together with general results on $h_n[w]$ in connection with Padé approximation and continued fractions for some formal Laurent series $\in K((z^{-1}))$ with coefficients coming from one word w, where K is the field of the rational functions $\mathbf{Q}[A]$.

Secondly, we refer to a new results related to the reducibility of $h_n[w]$ when w is

(3) a 2-adic Toeplitz word,

and then give some problems.