講演趣旨 解析数論研究集会@ RIMS Oct. 2004

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In this talk we show that integration of the non-holomorphic Eisenstein series for the full modular group with respect to certain specific non-negative measures $\mu(z)$ gives meromorphic functions $F_\mu(s)$ with poles at $s = 0, 1$ that have all their zeros on the line $\mathrm{Re}(s) = \frac{1}{2}$.

In particular the Riemann hypothesis is shown valid for all Arthur truncation integrals for $T \geq 1$. At the value $T = 1$ this proves the Riemann hypothesis for a zeta function $Z_{2,\mathbf{Q}}(s)$ recently introduced by Lin Weng, associated to rank 2 semistable lattices over $\mathbf{Q}$.

In addition the Riemann hypothesis is valid for the constant term $a_0(y, s)$ of the Eisenstein series at $y = 1$; a modified Riemann hypothesis holds for all values $y \geq 1$, with at most two exceptional real zeros, which occur for those $y > 4\pi e^{-\gamma} = 7.0555+$.