

In general, the coefficients to the Padé approximants to algebraic functions increase explosively in height with the degree of the approximating polynomials. This is a bad thing for the purposes of diophantine approximation, but it makes it all the more interesting nonetheless to be able to extract arithmetic information from the continued fraction expansion of a quadratic irrational function defined over, say, \mathbf{Q} .

For example, the sequence defined by $A_{h-2}A_{h+2} = A_{h-1}A_{h+1} + A_h^2$ and $A_{-1} = A_0 = A_1 = A_2 = 1$ arises from the curve $V^2 - V = U^3 + 3U^2 + 2U$ by reporting the denominators of the points $M + hS$, with $M = (-1, 1)$ and $S = (0, 0)$. The recursion $B_{h-3}B_{h+3} = B_{h-2}B_{h+2} + B_h^2$ and $B_{-2} = B_{-1} = B_0 = B_1 = B_2 = B_3 = 1$ arises from adding multiples of the divisor at infinity on the Jacobian of the curve $Y^2 = (X^3 - 4X + 1)^2 + 4(X - 2)$ of genus 2 to the divisor given by $[(\varphi, 0), (\bar{\varphi}, 0)]$; here it will please adherents to the cult of Fibonacci to learn that φ is the golden ratio.