NEW SERIES TRANSFORMATIONS FOR EULER'S CONSTANT

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Abstract

Let $s_n = 1+1/2+\ldots+1/(n-1)-\log n$. In 1995, the author has found a series transformation of the type $\sum_{k=0}^{n} \mu_{n,k,\tau} s_{k+\tau}$ with integer coefficients $\mu_{n,k,\tau}$, from which geometric convergence to Euler's constant γ for $\tau = \mathcal{O}(n)$ results. In recently published papers *T.Rivoal* and *Kh. and T.Hessami Pilehrood* have generalized this result. In the lecture a series transformation $\sum_{k=0}^{n} \mu_{n,k,\tau_1} s_{k+\tau_2}$ with two parameters τ_1 and τ_2 satisfying $\tau_1+1 \le \tau_2 \le n+\tau_1+1$ and integer coefficients μ_{n,k,τ_1} will be introduced. By applying the Mellin-Barnes integral representation of the ${}_3F_2$ - function, combinatorial identities and the analysis of the ψ-function, for $n = 2m$, $\tau_1 = m-1$ and $\tau_2 = 2m$ it is shown that $S := |\sum_{k=0}^{n} \mu_{n,k,\tau_1} s_{k+\tau_2} - \gamma| \le m/2 \cdot |\zeta(2) - q_m|$, where q_m are explicitly given rational numbers. Finally, $\zeta(2) - q_m$ can be expressed in terms of Legendre-type integrals, which give upper bounds for S. In particular, for $n = 2m$, $\tau_1 = m-1$ and $\tau_2 = 2m$ this bound equals to $2m \cdot 64^{-m}$. By the way, we find a linear three-term recurrence formula for a specific weighted sum on a ${}_3F_2$-function.