Fundamental properties of cellular automata and computation ability

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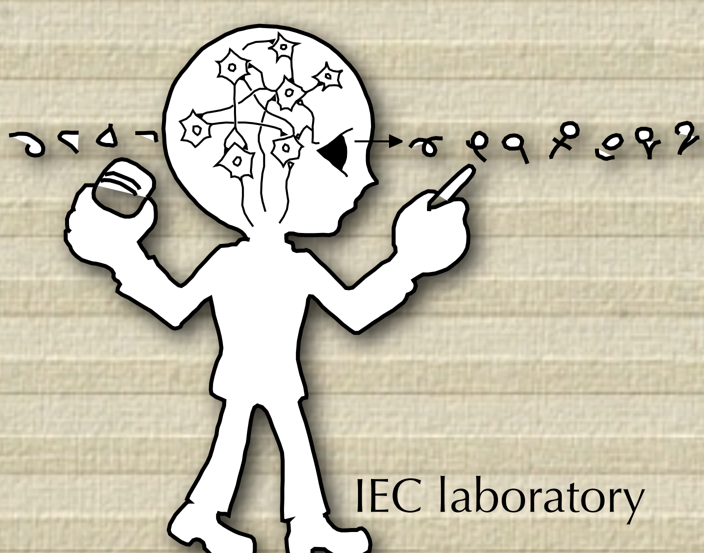


IEC laboratory

Quasi periodic tile and language theory (Kyoto) Jun. 8-10 2009

Remark:

Prof. Akiyama asked me to make a survey presentation of cellular automata.

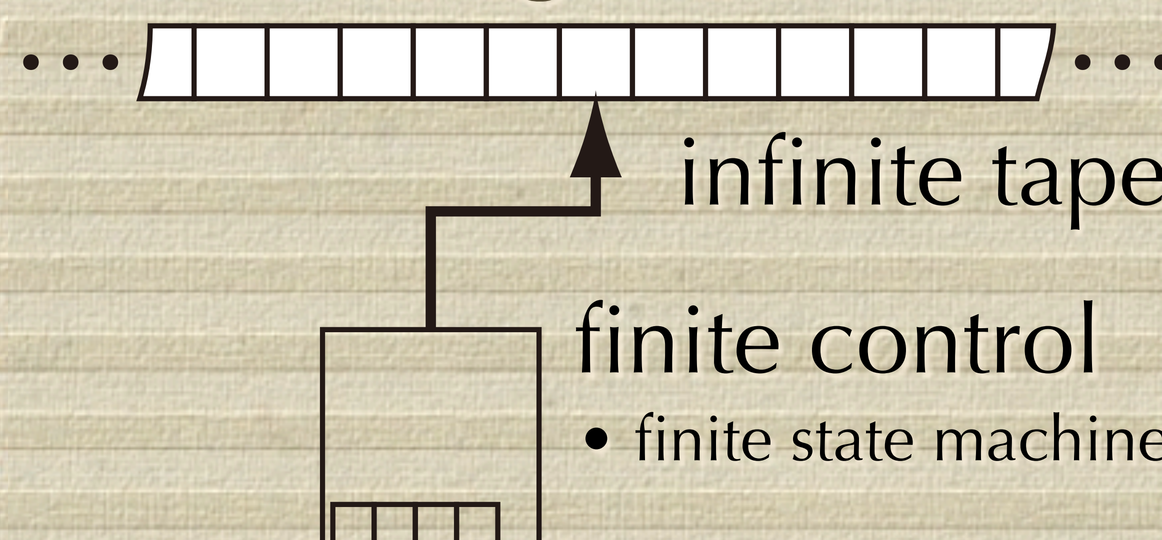
But impossible for me!

Today, I just focus on universality of cellular automata.

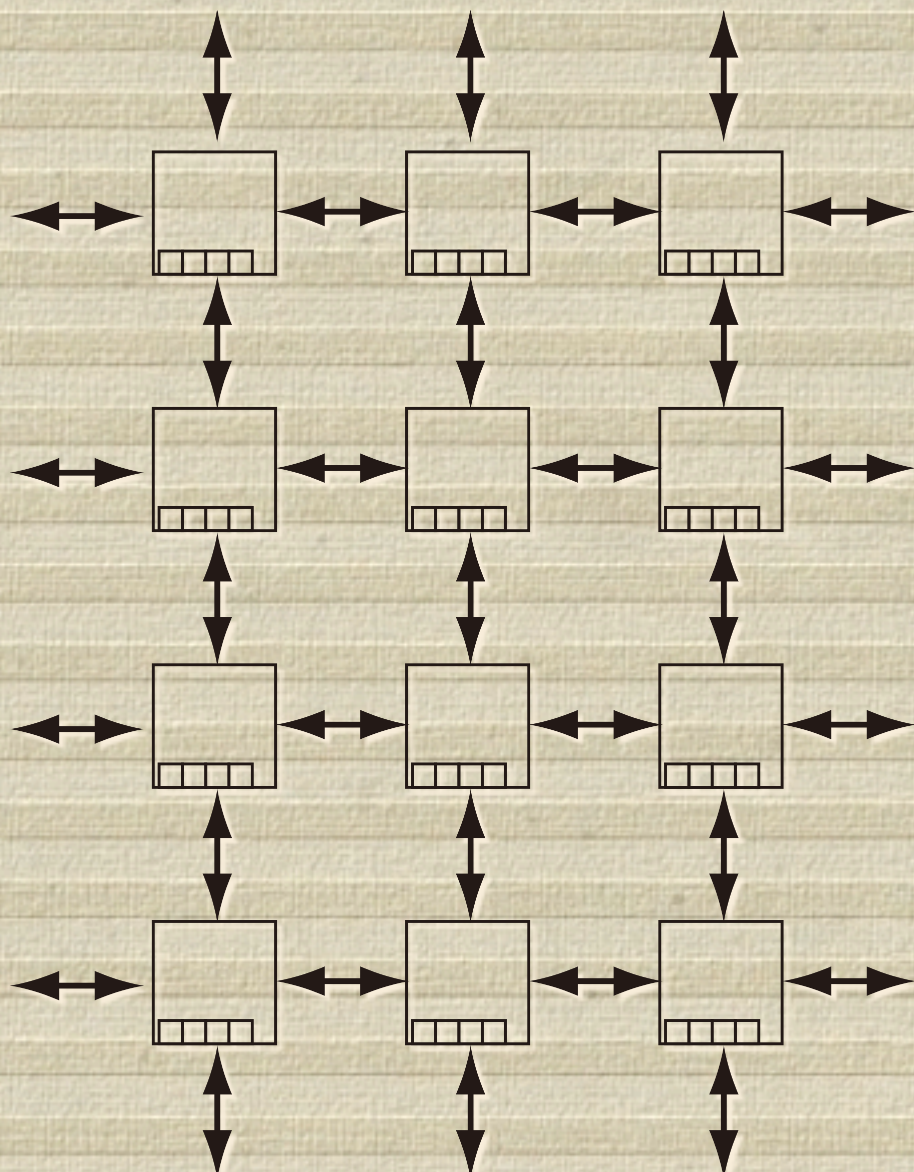
Automata

- A computing model with finite memory

cf. Turing machine



Cellular Automata (CA)



- A massively parallel computing model
  - systolic arrays
  - connection machines
- A physical model
  - car traffics
  - fluids
  - universe???

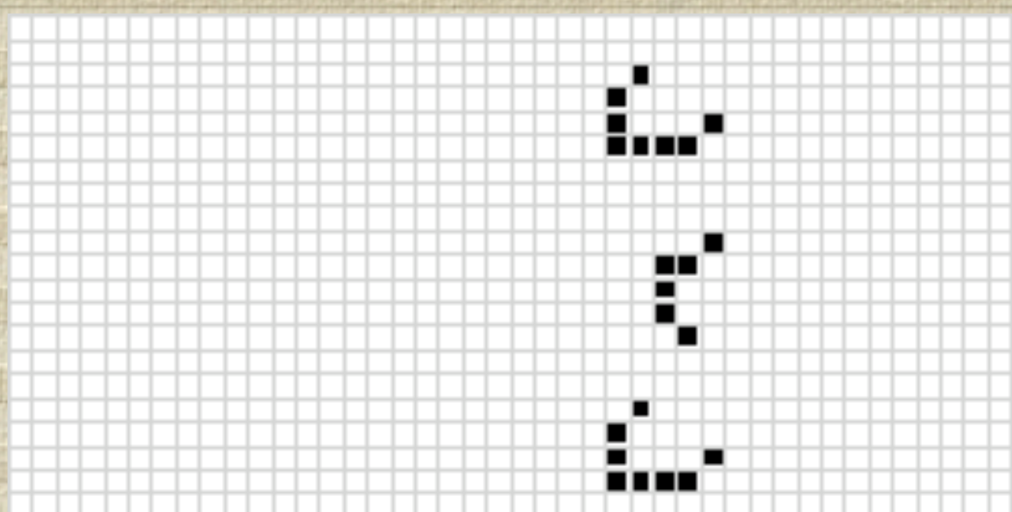
I have no idea.
Just ask to Fredkin or Wolfram.

cf. subshifts in symbolic dynamics

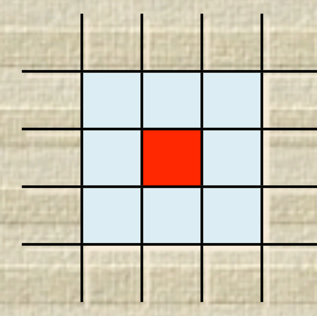
The game of “Life”

The most famous cellular automaton

Conway 1970

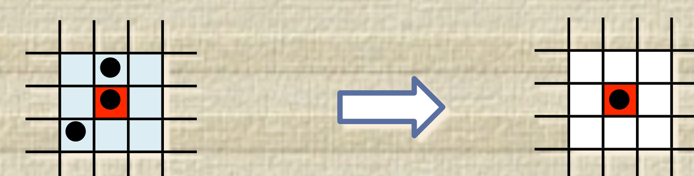


http://mathworld.wolfram.com/CellularAutomaton.html



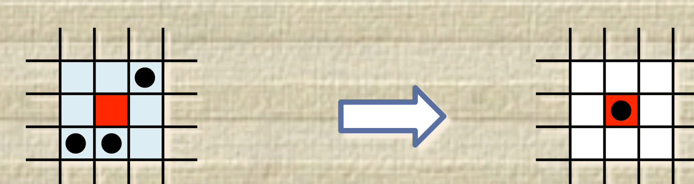
Moore neighborhood

Survive



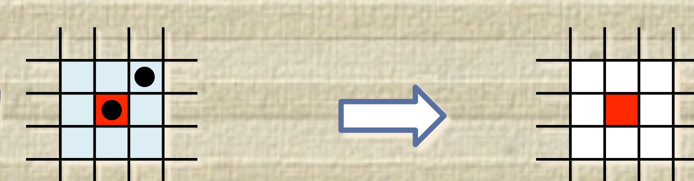
2 or 3 alive cells around

Birth



3 alive cells around

Death



otherwise

Birth range [3,3]

Survive range [3,4]

Universality in Cellular Automata

von Neumann 1950's

- computation-universality
  - You can 'compute anything' by a machine.
- construction-universality
  - You can 'construct anything' by a machine.
    (including the computing machine itself)

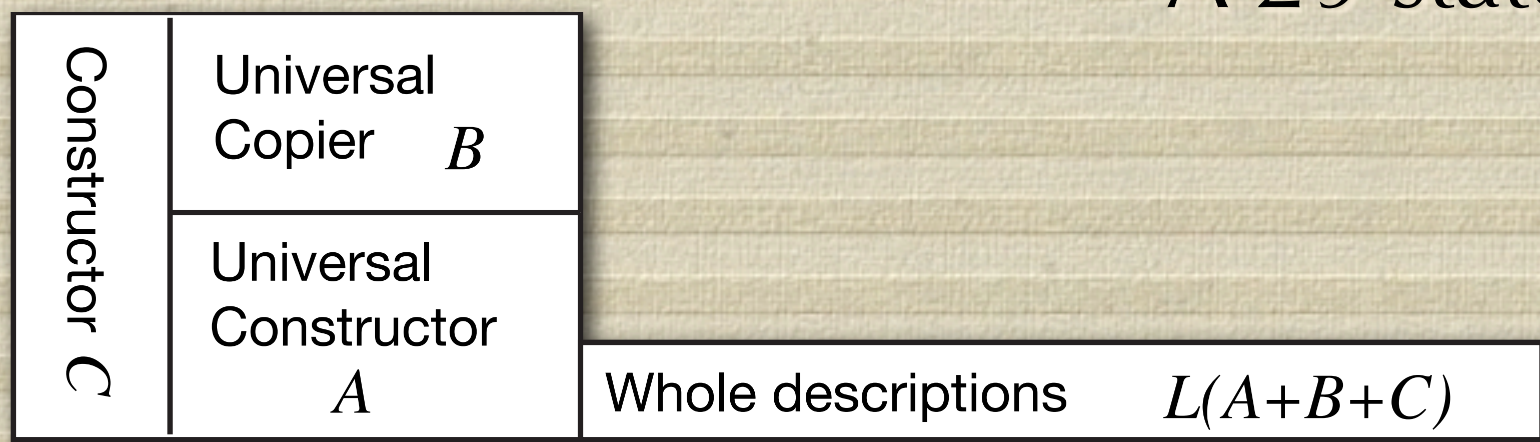
The stages of self-reproducing models by von Neumann

- kinematic model (-1948)
- cellular model ⇐ Ulam(-1953)
- excitation-threshold-fatigue model
- continuous model by differential equation
- probabilistic model mutation

crystal growth < ・・・ < biological self-reproduction

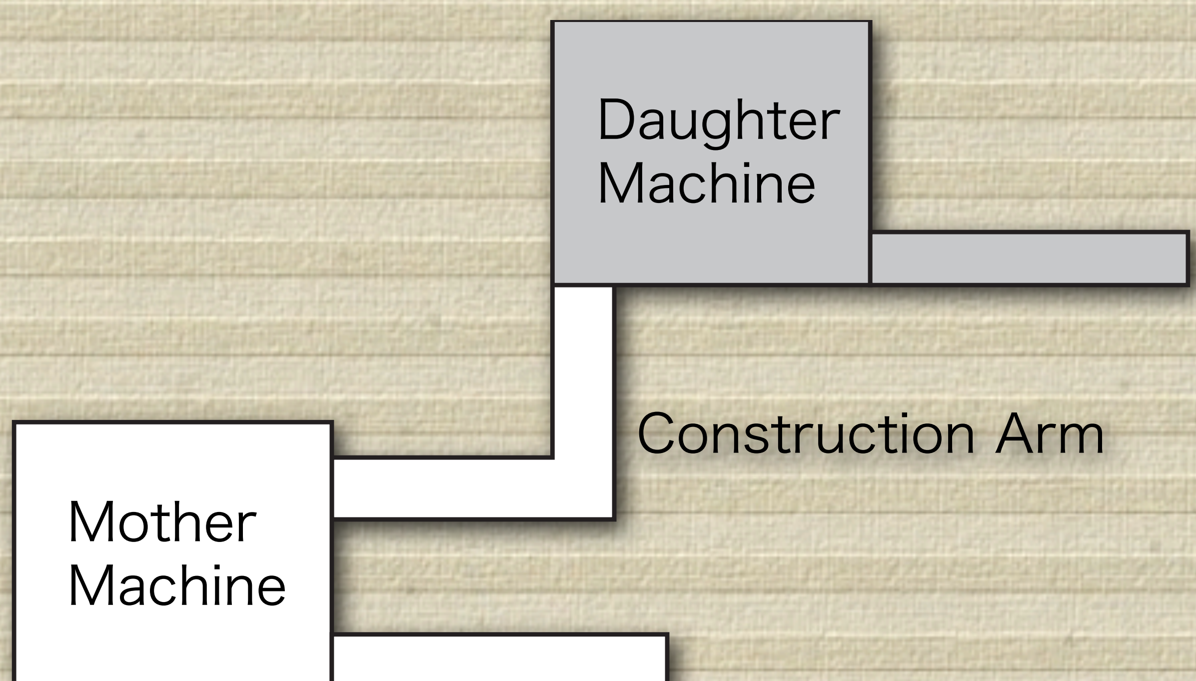
A self-reproducing celluar automaton by von Neumann

A 29 state cellular automaton



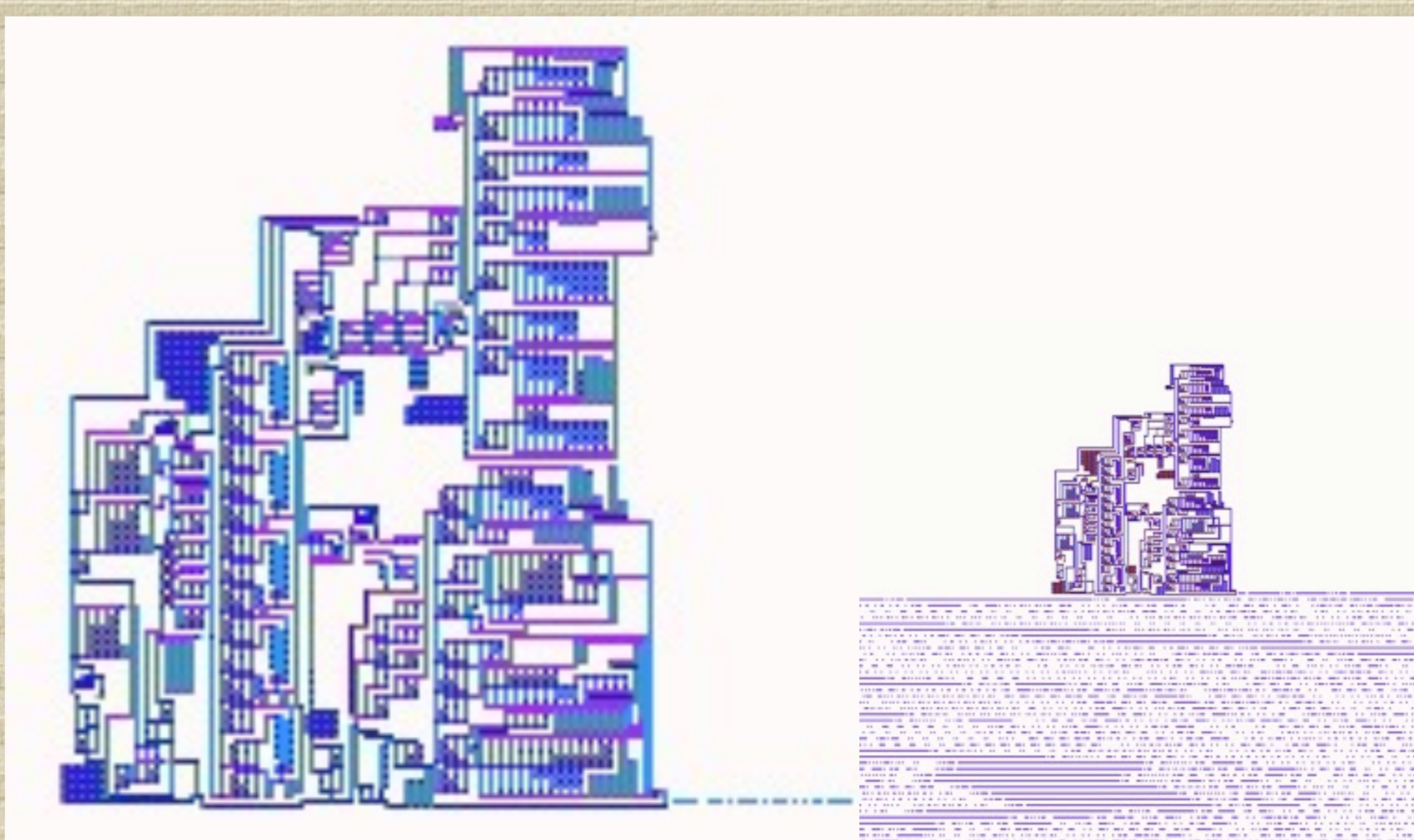
von Neumann 1950-

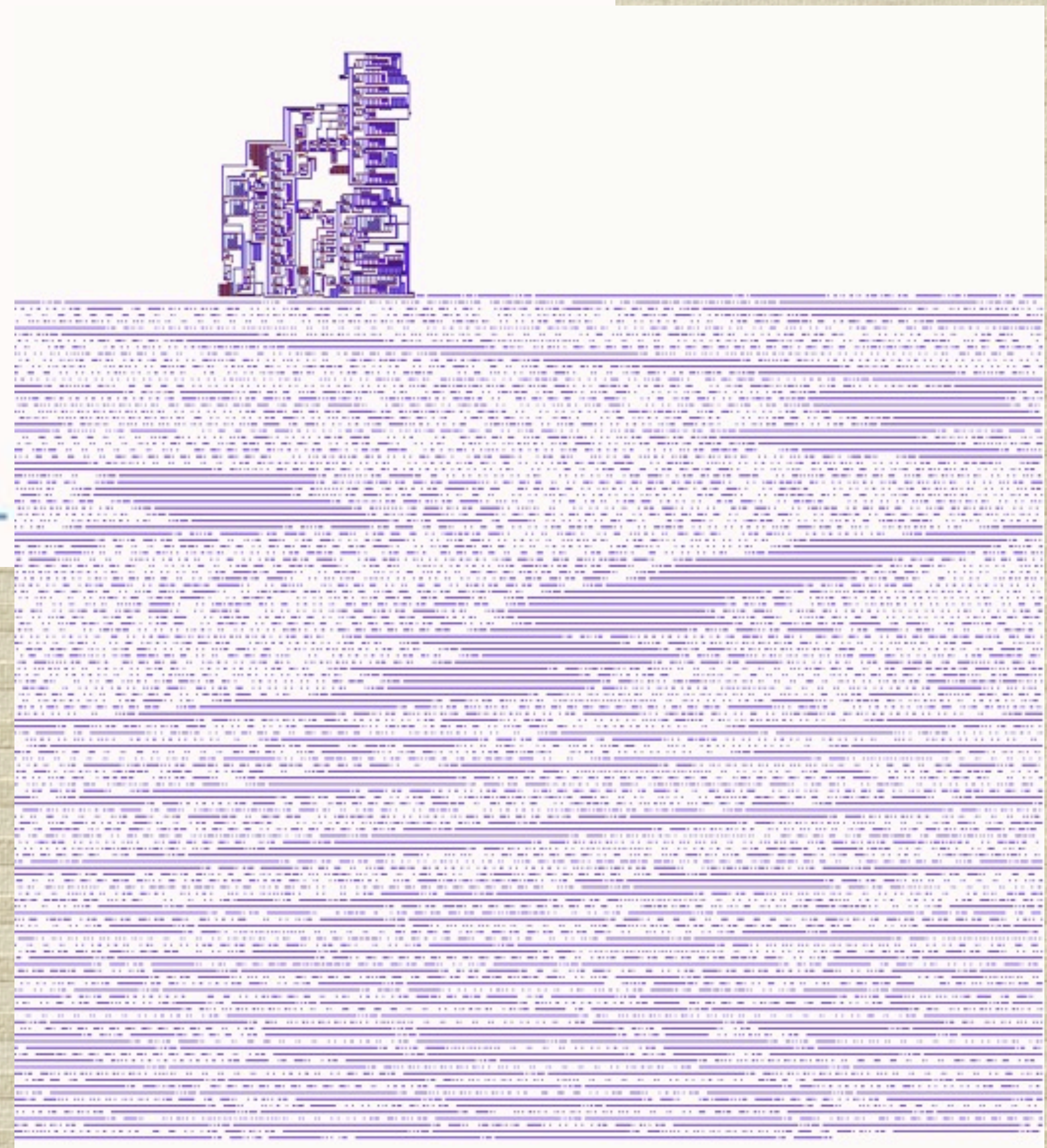
(cf. Watson, Crick 1953)



Computation and construction universal

von Neumann (Burks ed.)1966





(Nobili-Pesavento 1996)

Definition

A deterministic cellular automaton $A = (d, Q, N, f)$

dimension: d finite set of states: Q (quiescent state: $q \in Q, f(q, q, q) = 0)$

local function: $f : Q^N \to Q$

configuration: $\mathrm{Conf}(Q) = \{\alpha | \alpha : \mathbf{Z}^d \to Q\}$

neighborhood: $N = \{\nu_1, \cdots, \nu_n\}$

global function:

$$F : \mathrm{Conf}(Q) \to \mathrm{Conf}(Q)$$

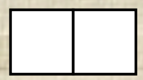
$$\forall z \in \mathbf{Z}^d,$$

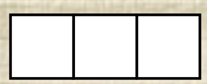
$$F(\alpha)(z) = f(\alpha(z + \nu_1), \alpha(z + \nu_2), \cdots, \alpha(z + \nu_n))$$

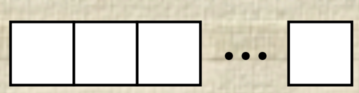
Neighborhoods

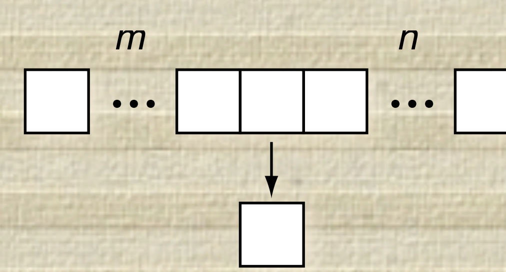
choose one as you like

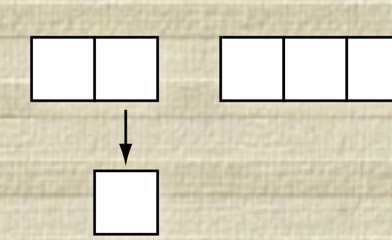
one dimensional case


radius 1/2, one-way

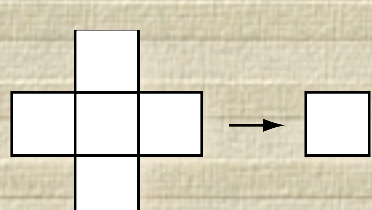

radius 1

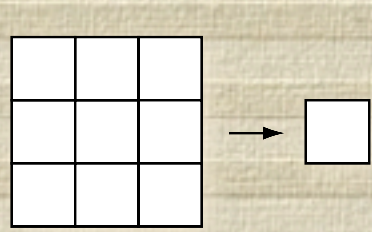


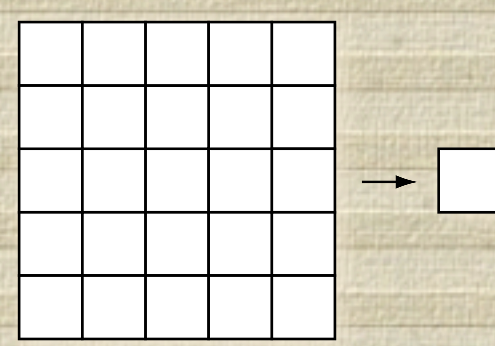


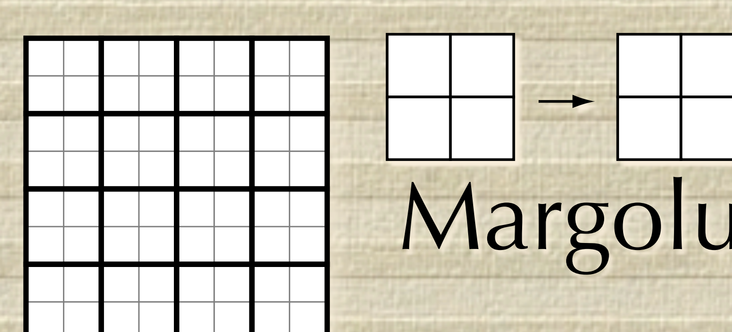


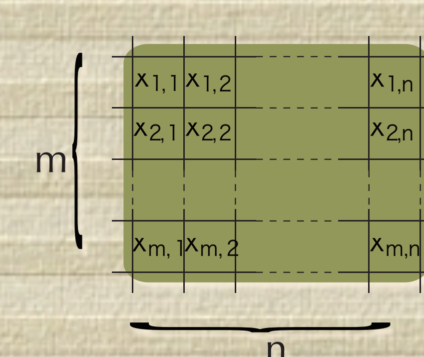
two dimensional case


von Neumann


Moore

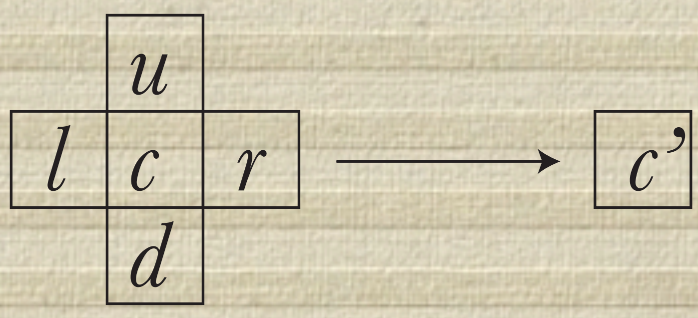

radius 2 Moore


Margolus



three or more dimensional case...

Local function



$$f(c, u, r, d, l) = c'$$

local function

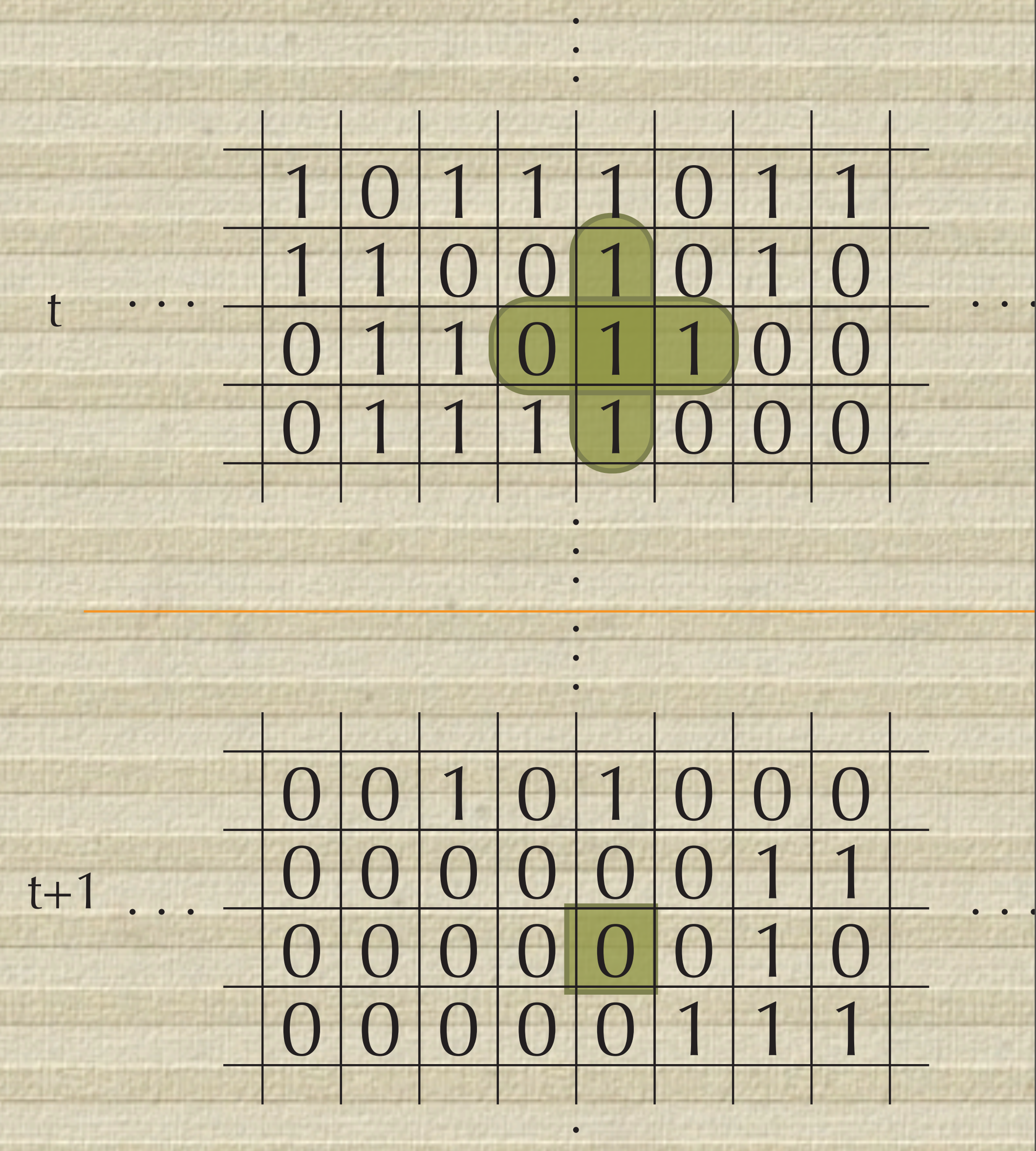
$$f(0,0,0,0,0) = 0$$

$$f(0,0,0,0,1) = 1$$

$$\vdots$$

$$f(1,1,1,1,0) = 0$$

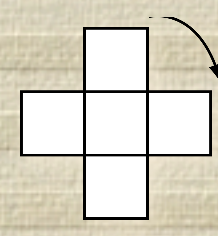
$$f(1,1,1,1,1) = 0$$



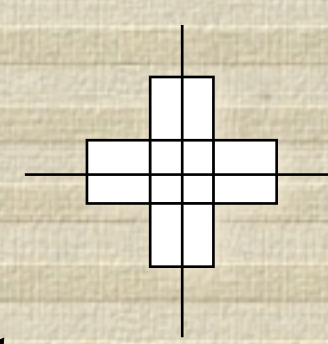
Frequently used symmetric conditions

The number of distinct rules is too large for the daily use...
It depends on the chosen shape of neighborhood.

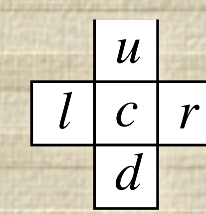
- rotation-symmetry



- reflection-symmetry



- totalistic, permutation-symmetry



- ...

$f(c,u,r,d,l)$ is independent of the order of u,r,d,l

The rule can be written as the sum of states in the case of $Q=\{0,1\}$

Finite and infinite configuration

It is reasonable to introduce infinite cellular space.

- Finite (initial) configuration:
  - the number of non-quiescent cells is finite
  - i.e., the “mass” of the cellular space is finite
- Infinite (initial) configuration
  - periodic (with a given constant period)
    - combination of infinite periodic background and a finite pattern
  - random (with a given distribution)
  - ...

‘fair’ setting for computation
cf. rule 110

Basic properties of global function

Defs.

CA is injective: its global function is injective.

CA is surjective: its global function is surjective.

CA is reversible: there is a CA that reverts it.

Garden of Eden

A configuration with no predecessor

Banks 1971

CA is surjective ⇔ CA has no Garden of Eden

CA is injective ⇒ CA is surjective

Global function restricted to finite configuration is injective ⇔ CA is surjective

Moore 1962, Myhill 1963

Reversible cellular automata

CA is reversible ⇔ CA is injective

Hedlund 1969, Richardson 1972

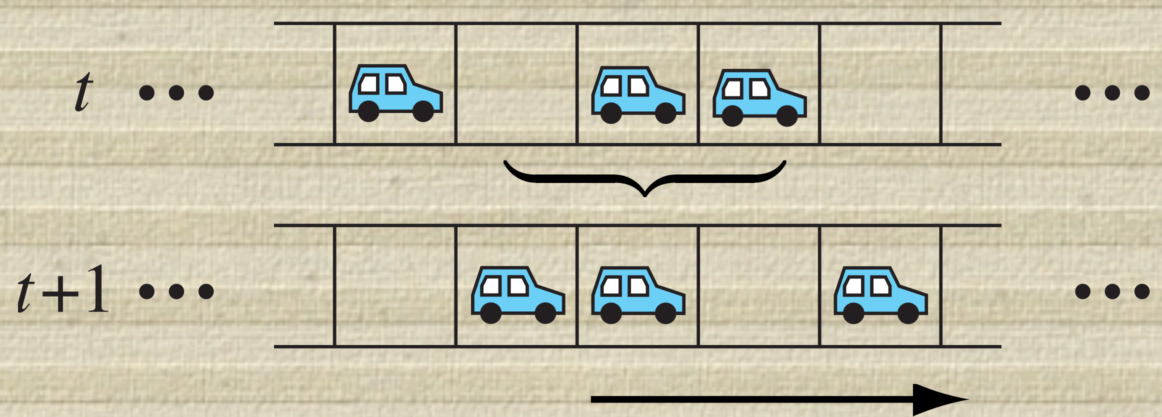
Reversibility (surjectivity) is decidable in one-dimensional case

Amoroso, Patt 1972

Reversibility (surjectivity) is undecidable in two (or more)-dimensional case

Kari 1990, 1994

Number (Momentum) Conserving Cellular Automata



$f(0,0,0)=0$
$f(0,0,1)=0$
$f(0,1,0)=0$
$\underline{f(0,1,1)=1}$
$f(1,0,0)=1$
$f(1,0,1)=1$
$f(1,1,0)=0$
$f(1,1,1)=1$

A simple car traffic rule
(Nagel, Schreckenberg 1992)

cf. lattice gases (Margolus neighborhood)

Number (Momentum) Conserving Cellular Automata

- 1991 Hattri, Takesue: additive conserved quantities in 1D CA
- 2000 Boccara, Fuks: 1D NCCA is characterized by motion representation
- 2001 Durand, Formenti, Roka: number-conservation is decidable in any dimension
- 2005 Bernardi et, al.: number-decreasing property is decidable in one-dimension but undecidable in two ore more dimension

Turing Universality

- Let the cellular automaton simulate one of the well known universal models of computation.

Turing machine, 2-register (counter) machine,
Chomsky type-0 grammar, 2-TAG system,
cyclic TAG system, ...

m-TAG System

Post 1943

$$(m, A, P)$$

m: deletion number (positive integer)

A: finite set of alphabet (including a haling alphabet)

P: production rule

ex. 2-tag system

A={a,b,c,H}

P: a --> ccbaH

b --> cca

c --> cc

Initial word: baa

acca

caccbaH

ccbaHcc

baHcccc

Hcccccca (halt).

1-TAG is decidable, 2-TAG is Turing-universal.

Wang 1963, Cocke, Minsky 1964

Turing-universal even under a kind of context-freeness

Cyclic TAG System (CTAG)

Cook 2004

- Alphabet is limited to 0,1
- Production rules are applied sequentially (do not need random access table-lookup mechanism)

CTAG $C_1 = (3, \{ Y, N \}, (\varepsilon, YN, YY))$

$Y \rightarrow \varepsilon$ (0)
$Y \rightarrow YN$ (1)
$Y \rightarrow YY$ (2)

	applied rule
NYY	0
YY	1
YYN	2
YNYY	0
NYY	1

no halting state

cf. CTAG with halting state
Morita 2006

Logical Universality

limited to two or more dimensional case

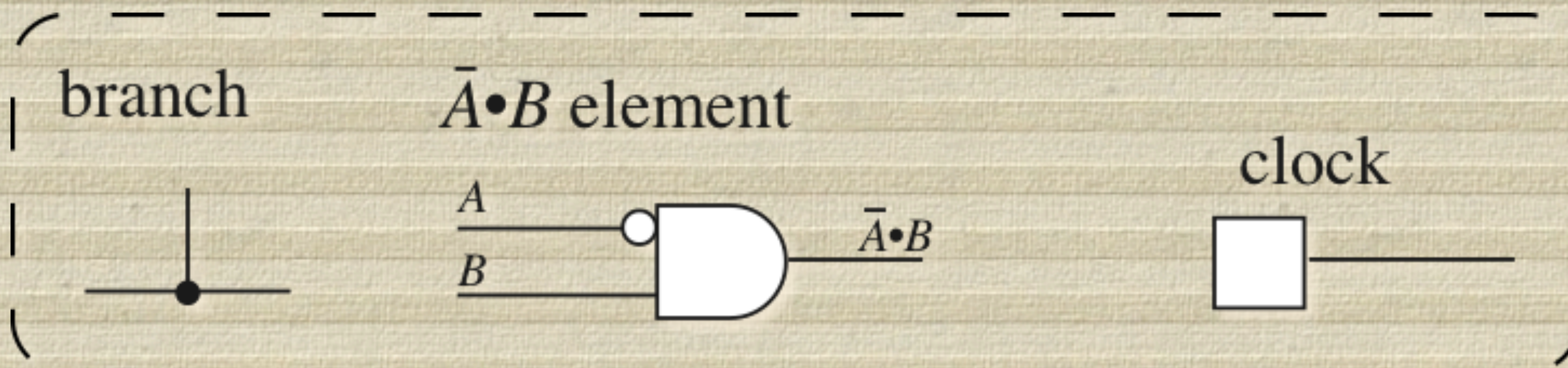
A cellular automaton is logically universal:

A set of universal logical elements and their wiring scheme are embedded to the cellular automaton.

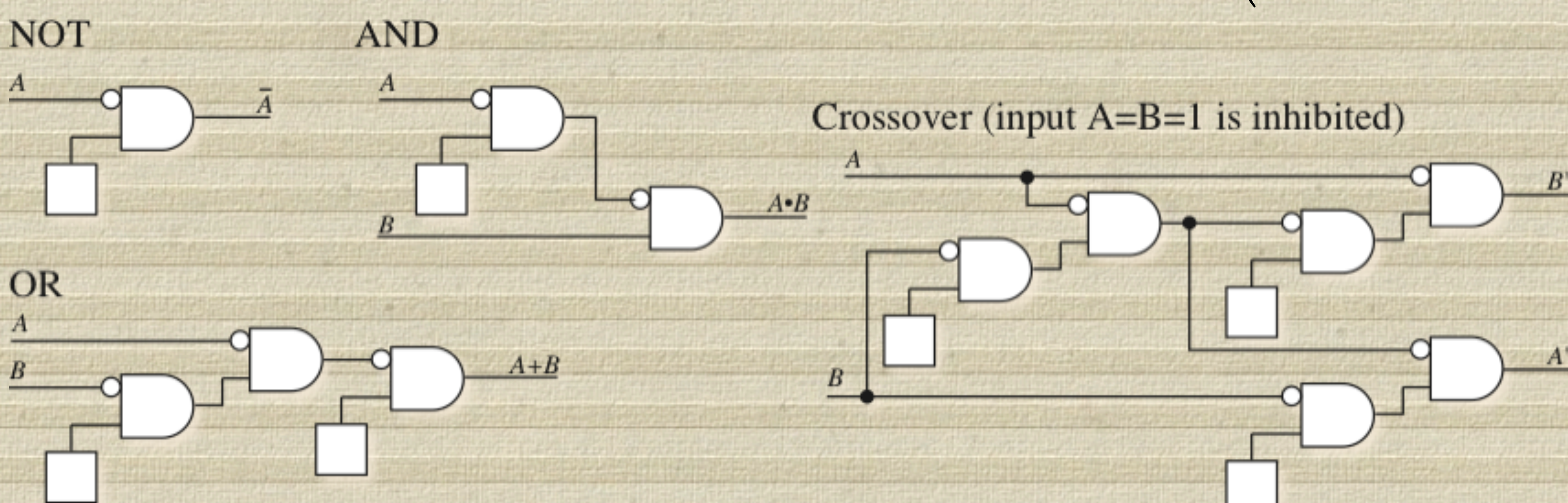
e.g. {NAND, unit delay}, {AND, OR, NOT, unit delay}

Any combinatorial circuit can be constructible.

Universal logic element set suitable for embedding in a CA



(Banks 1970)



Universal with Finite/Infinite Configuration

- Universal simulation can start with finite initial configuration
  - e.g. counter machine simulation by the game of Life
- Universal simulation can start with infinite but periodic initial configuration and a finite pattern
  - e.g. rule 110

Intrinsic universality

A universal Turing machine which simulates any other Turing machine.

A cellular automaton which simulates any other cellular automaton.

- cellular automata simulated by cellular automata in a shift invariant, time invariant way.

Roughly speaking...

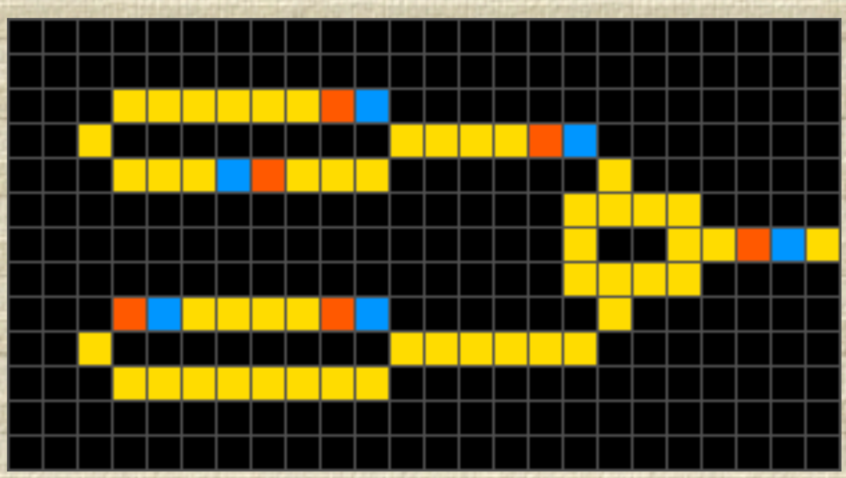
- logically universal CAs require infinite configurations
- In two or more dimensional case, logically universal $\Rightarrow$ intrinsic universal.
- logically universal $\Rightarrow$ Turing-universal
- construction-universal (in von Neumann's sense)
  - Computations can start with a finite configuration but the CA can construct the pattern of a Turing machine including an infinite periodic tape-structure.

2D von Neumann neighborhood Universal CA

- 1966 von Neumann 29-state, construction-universal
- 1968 Codd 8-state
  - construction-universal
  - proof of non-universality of 2-state finite-configuration
- 1970 Banks
  - 4-state construction-universal
  - 3-state finite configuration
  - 2-state infinite configuration
- 1983 Serizawa 3-state construction-universal

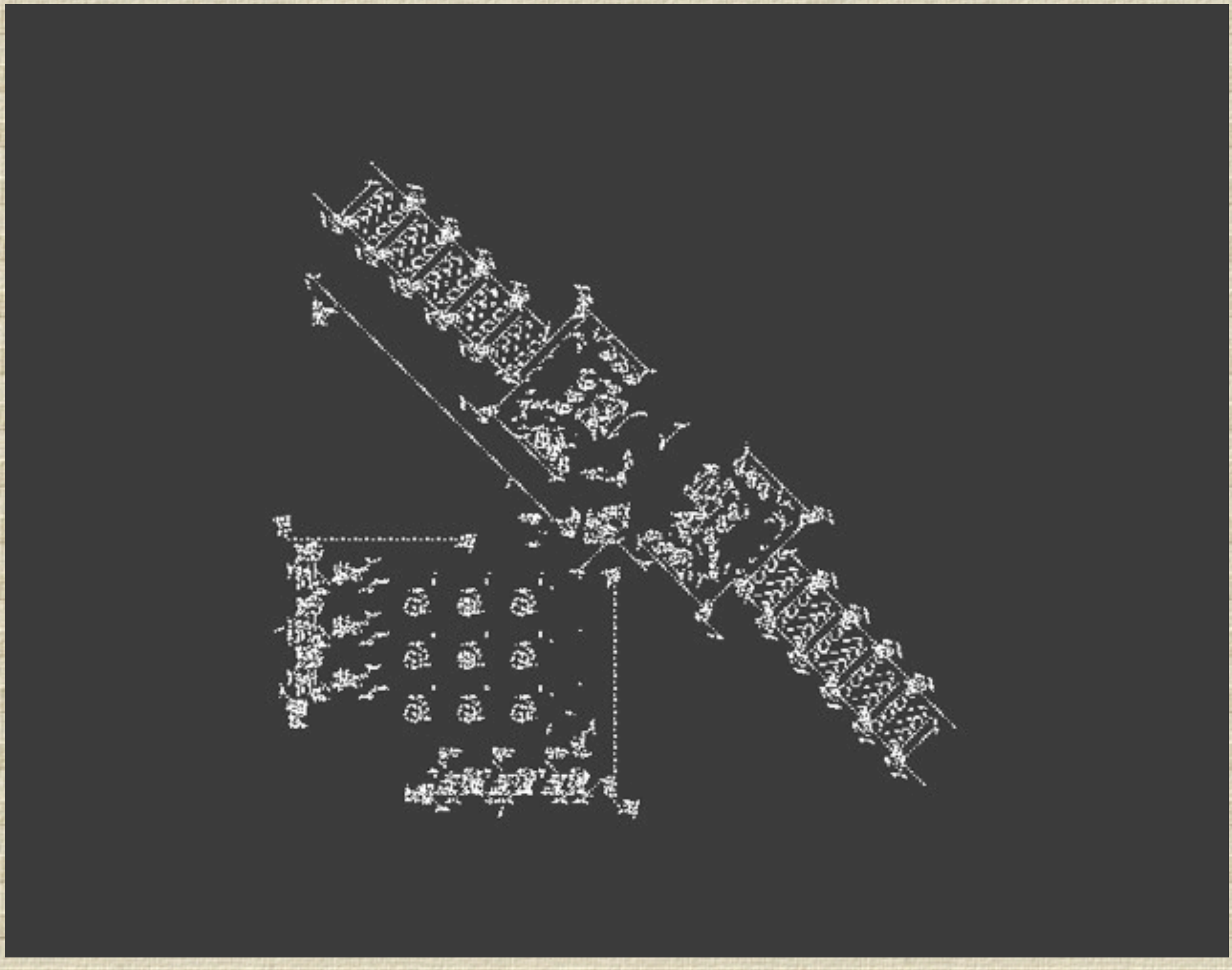
2D Moore neighborhood universal CA

- 1970 Berlekamp, Conway, Guy: 2-state, totalistic, construction-universal
- (1987 Silverman: Wireworld 4-state, logically-universal)



A realization of XOR gate in Wireworld (Wikipedia)

Direct embedding of Turing machine to the game of Life



(Rendell 2002)

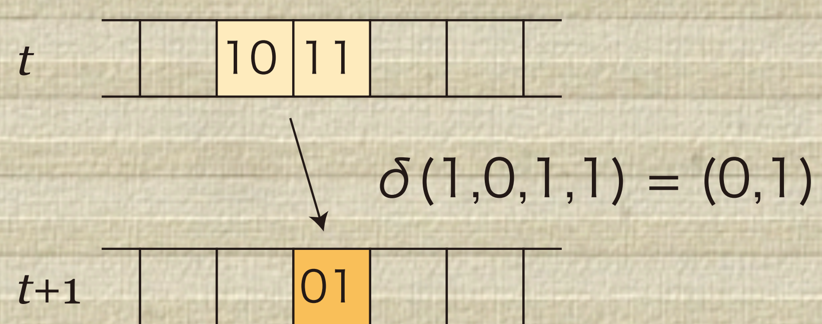
1D universal CA (radius 1)

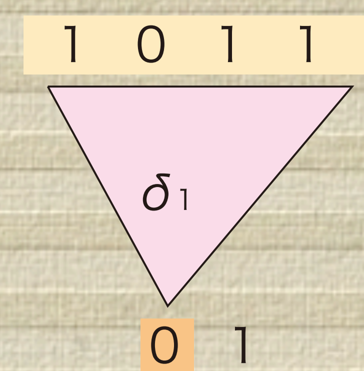
- 1971 Smith III: 18-state (simulation of TM)
- 1990 Lindgren, Nordahl: 7-state (simulation of TM)
- 2002 Olinger: Intrinsically-universal, 6-state
- 2004 Cook: 2-state (rule 110) CTAG
- 2008 Richard: Intrinsically-universal, 4-state

One-dimensional Intrinsic-universal CA

Olinger 2002

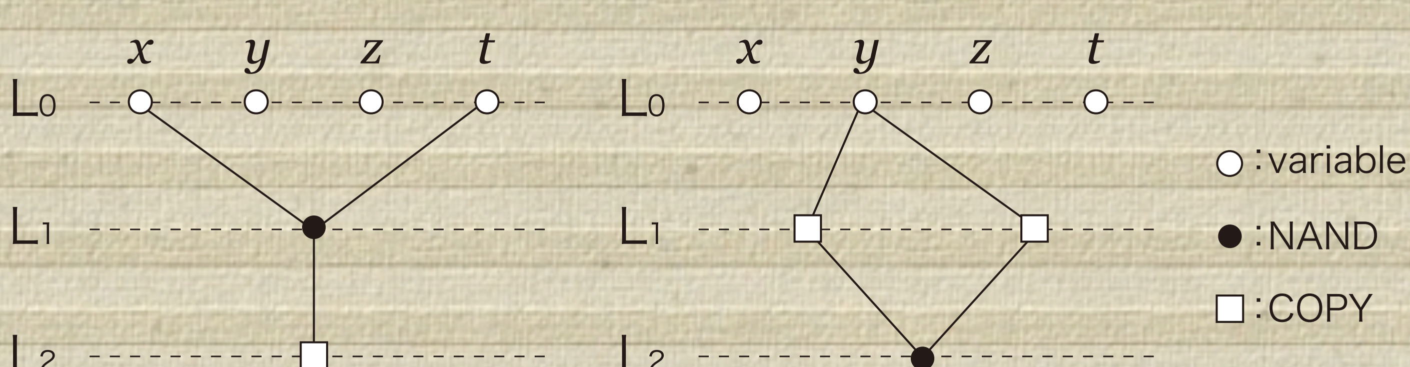
Each bits of binary encoded state value can be computed by a circuit made by NAND and COPY gates.



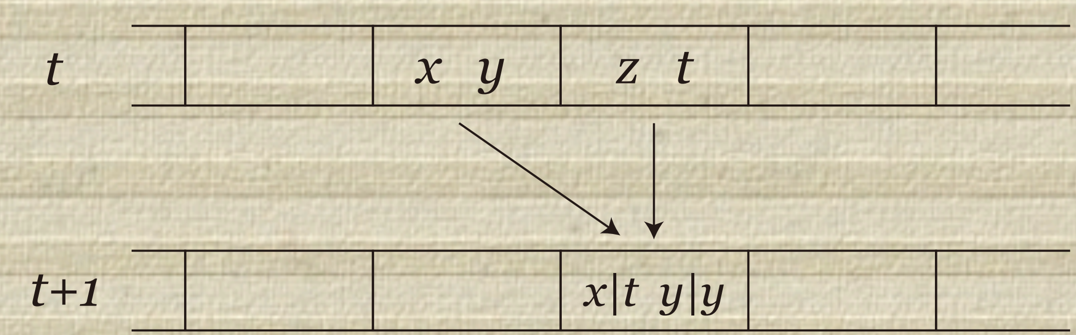


$\delta_1(1,0,1,1) = 0$

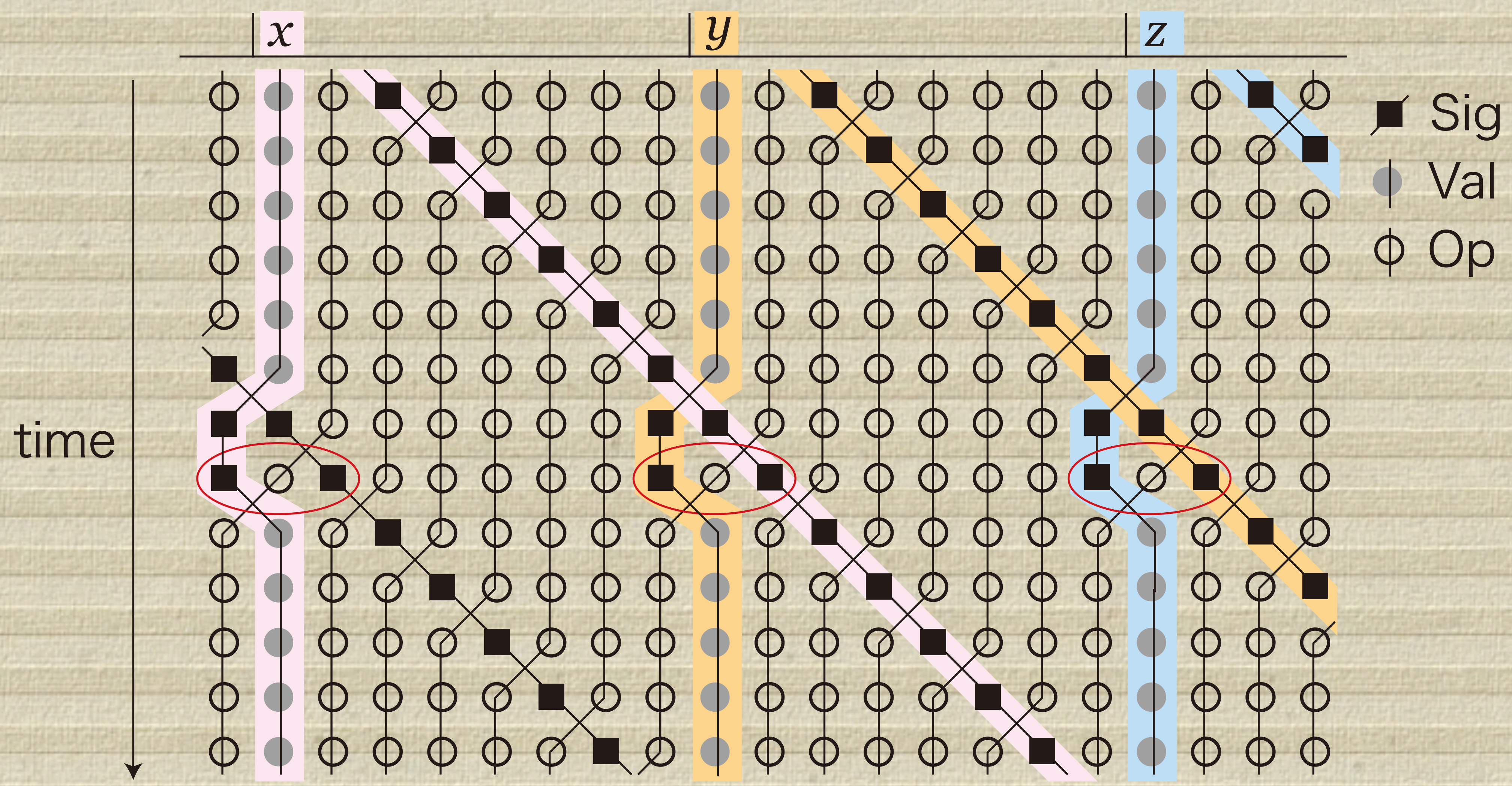
$\delta_2(1,0,1,1) = 1$



$\delta_1(x, y, z, t) = x|t$ $\delta_2(x, y, z, t) = y|y$



Ontline of a Cofiguration of Olinger's intrinsic-universal CA

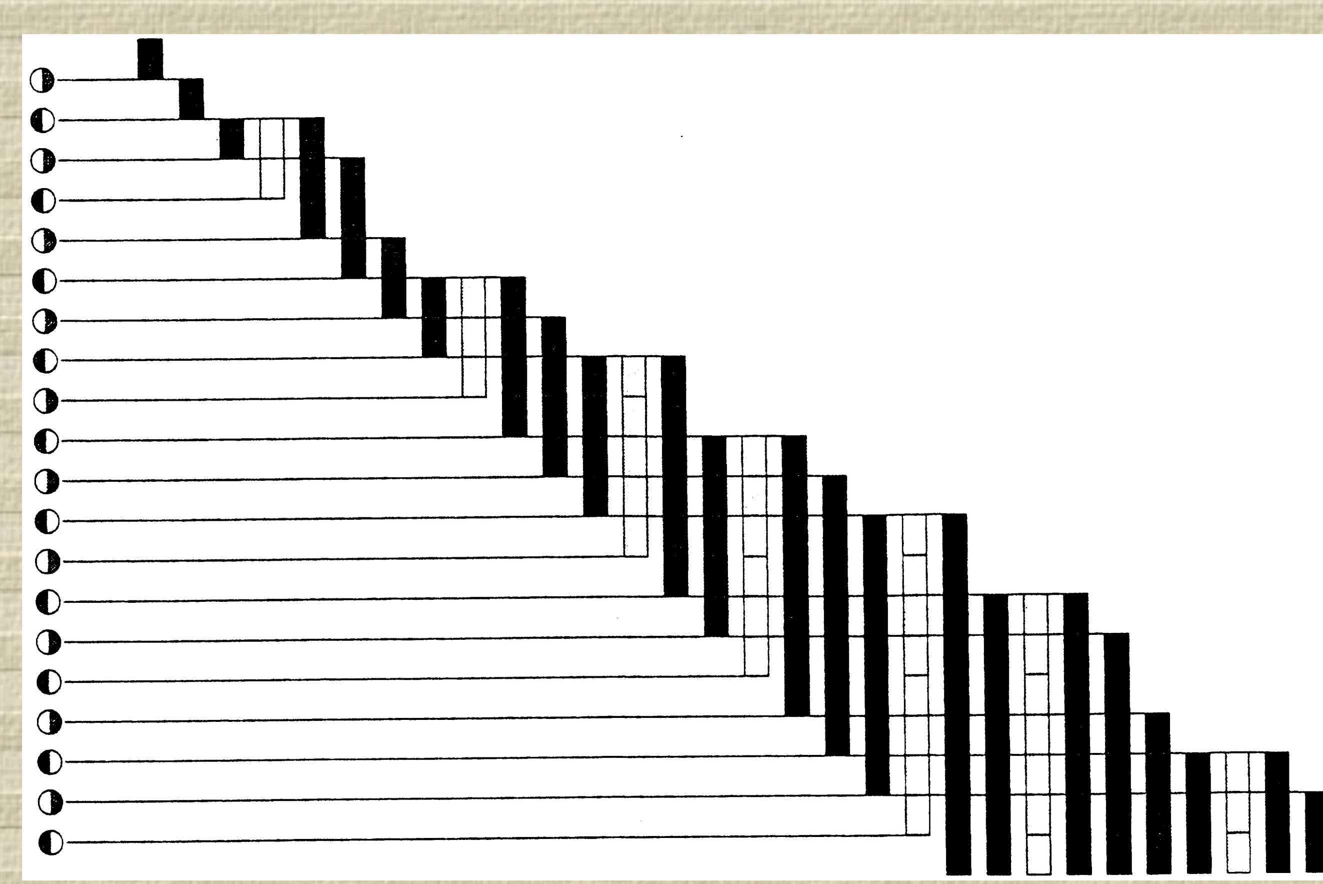


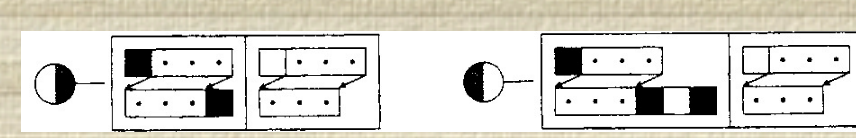
Universality of Rule 110

Cook 2004

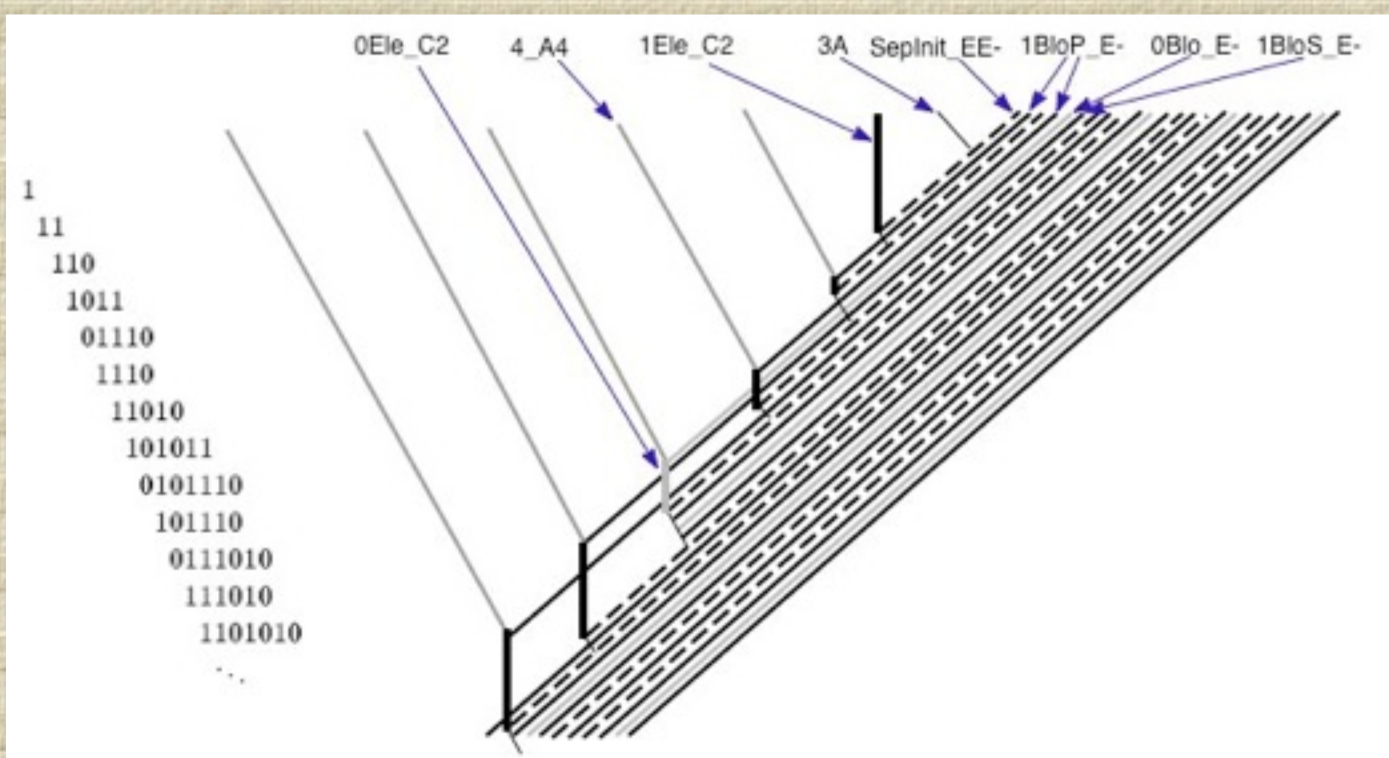
Simulation of CTAG

000	001	010	011	100	101	110	111
0	1	1	1	0	1	1	0

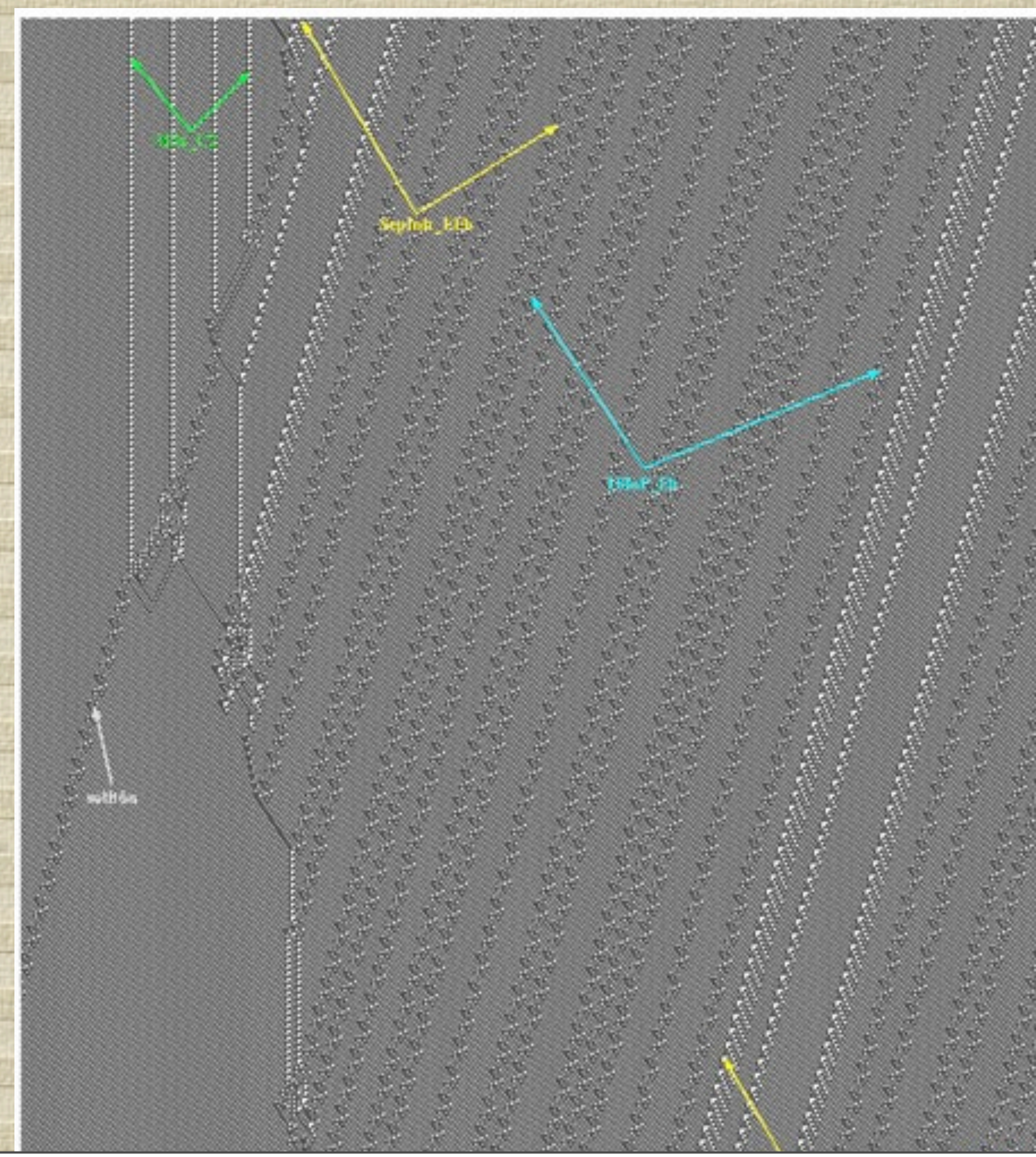




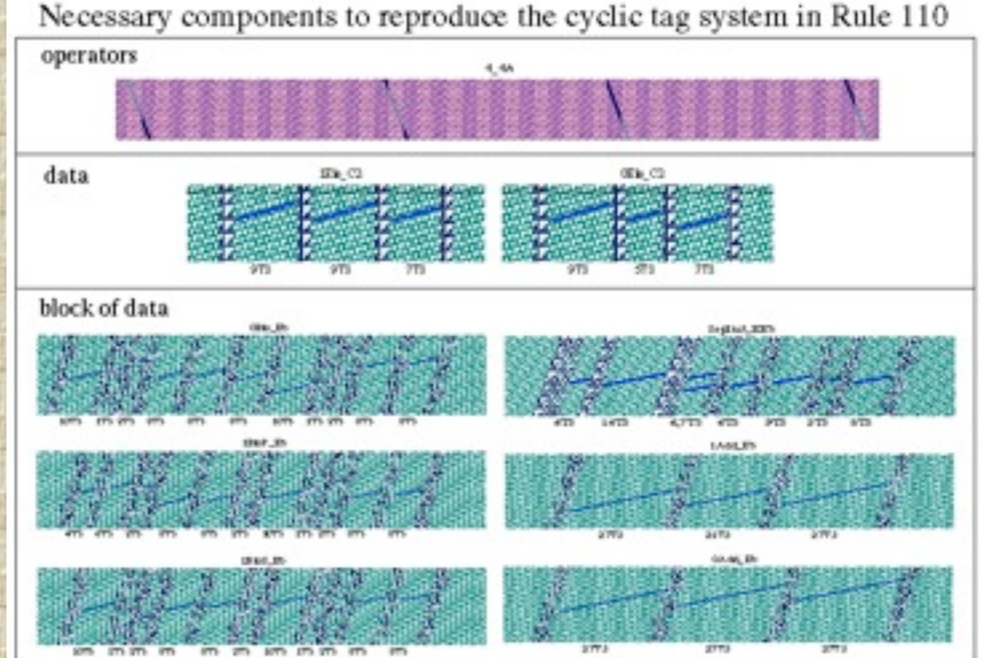
Outline of space time diagram of CTAG simulation by Rule 110



Cook 2004 cf. Martinez 2006



Necessary components to reproduce the cyclic tag system in Rule 110

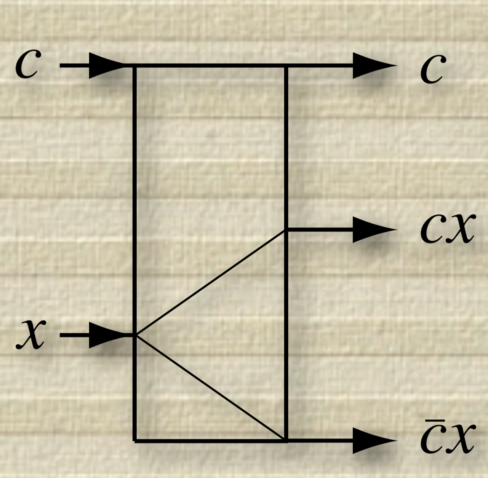


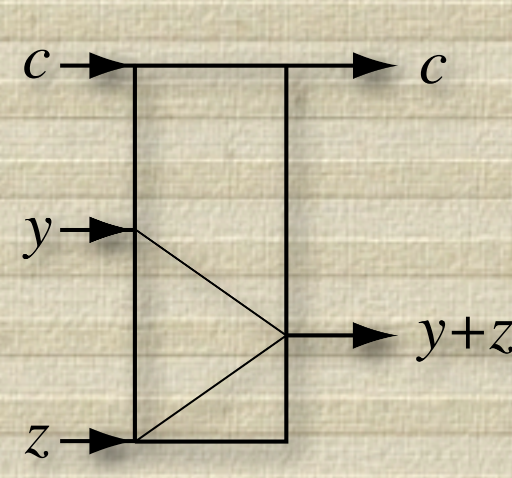
Universal Reversible CA

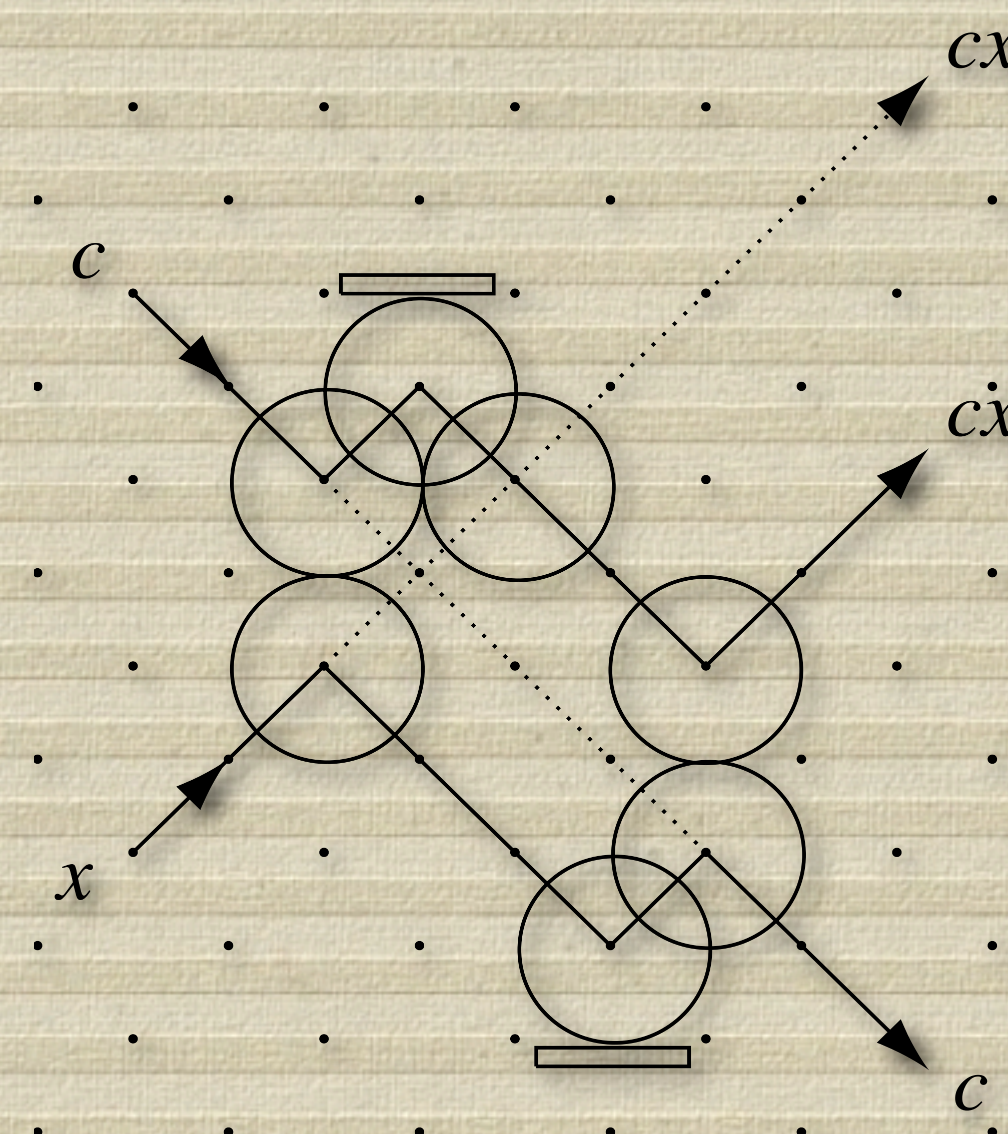
- 1977 Toffoli: Turing-universality of 2D reversible CA
- 1984 Margolus: 16-state, 2D, billiard ball model CA, logically-universal
- 1989 Morita, Harao: 1D, simulation of reversible TM
- 1997 Durand-Lose: 1D, a weak sense of intrinsic-universal
- 2000 Imai, Morita: 8-state, 2D triangular neighborhood
- 2000 Morita, Ogiro.: 81-state, 2D von Neuman, finite-configuration
- 2008 Morita: 24-state, 1D radius 1, reversible CTAG
  - 2006 Morita: 98-state, 1D, radius 1, finite configuration, reversible CTAG with halting state

Reversible system can be logically-universal

Fredkin, Toffoli 1982





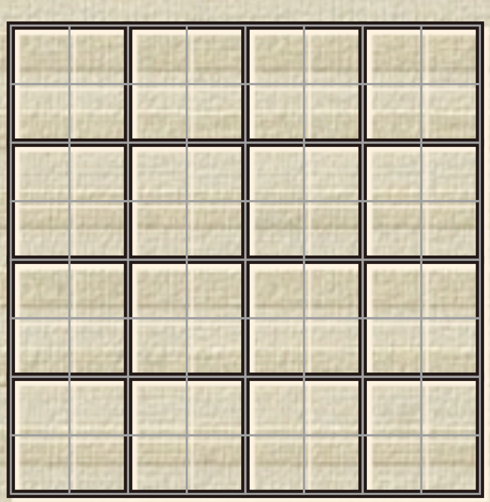


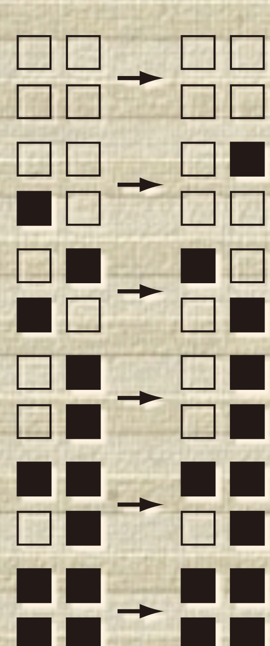
The BBM Cellular Automaton

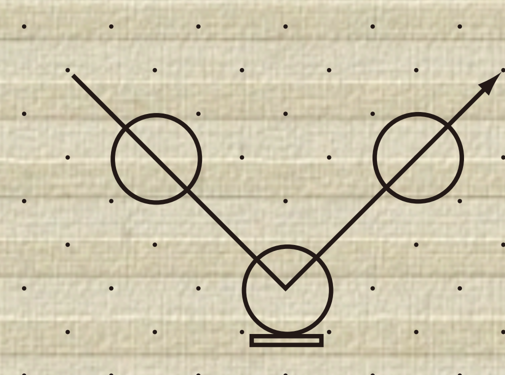
Reversible and logically-universal

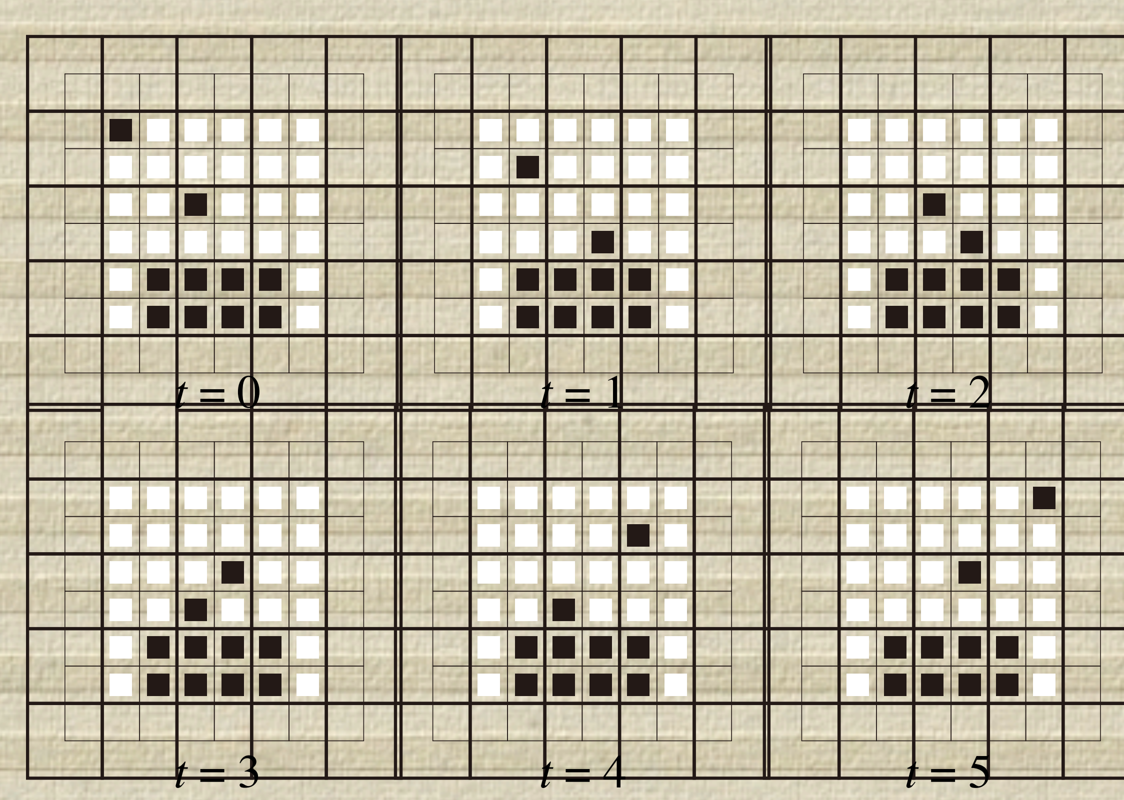
Margolus 1984

Margolus neighborhood

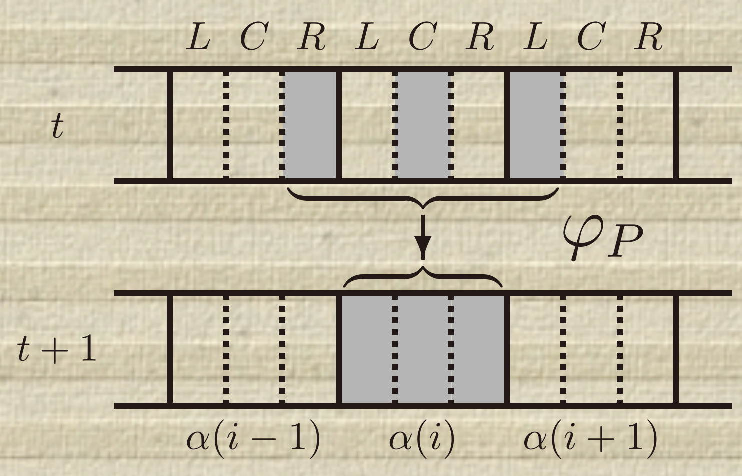








Reversible Partitioned CA



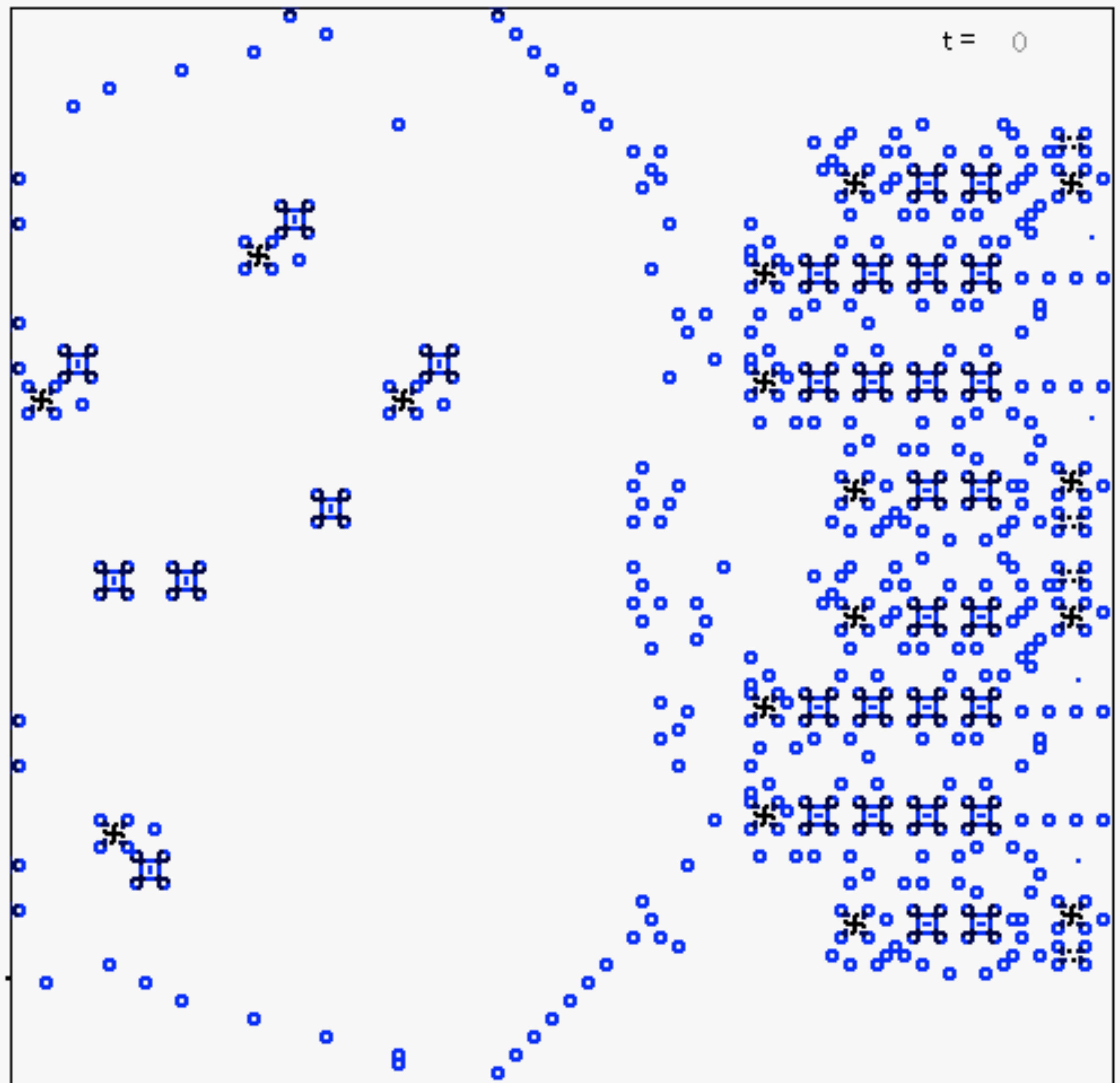
Morita, Ueno 1989, Morita, Harao 1989

- Each cell is partitioned into the equal number of parts to the neighborhood size
- A partitioned CA is reversible iff its local function is one-to-one
- PCA is a subclass of 'normal' CA

81-state 2 dimensional universal reversible partitioned CA

Simulation of two-counter machine

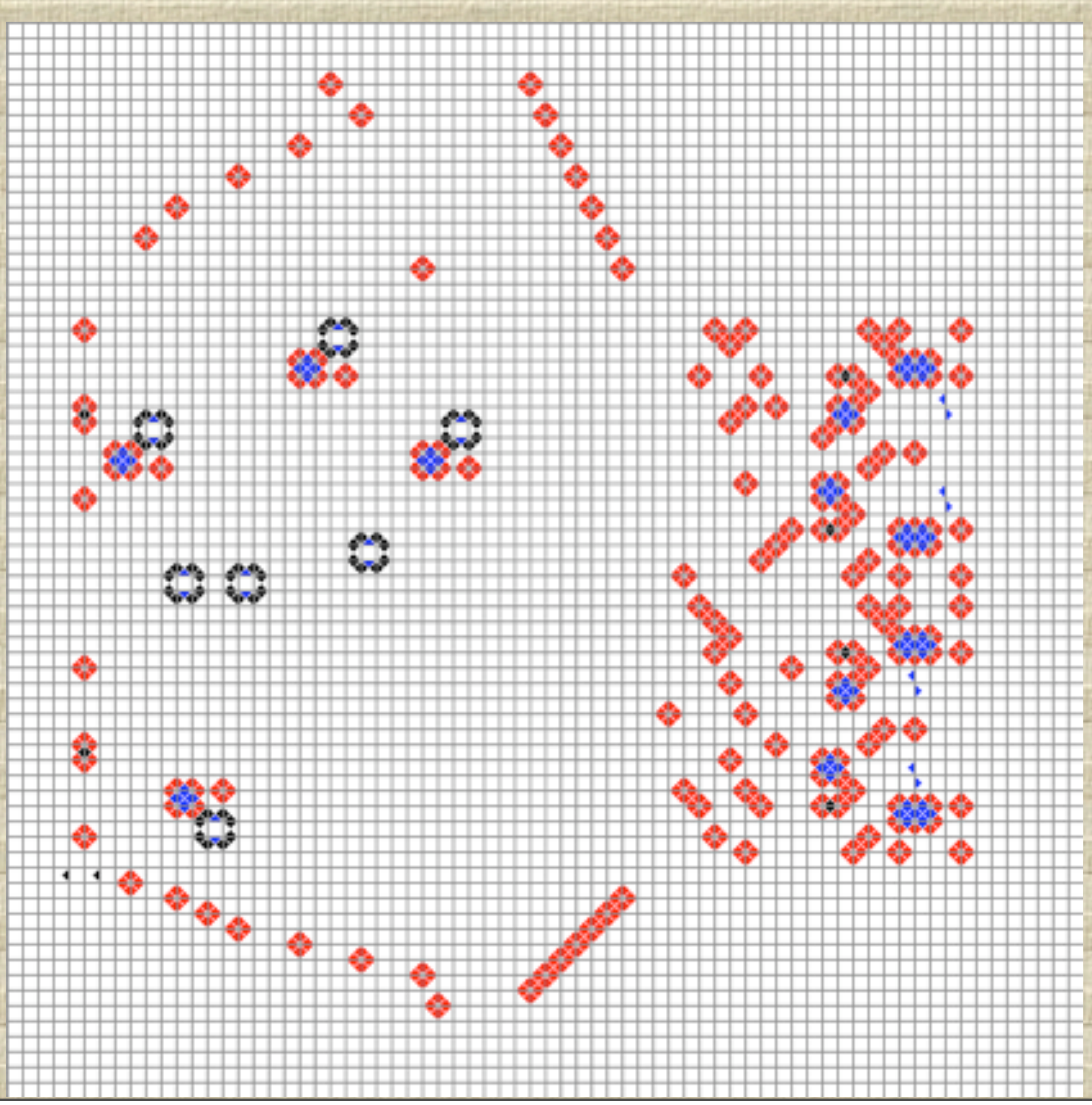
Morita, Ogiro 2000



An RCM(2) that computes y = 2x+2

3^4 – state

cf. 4^4 – state

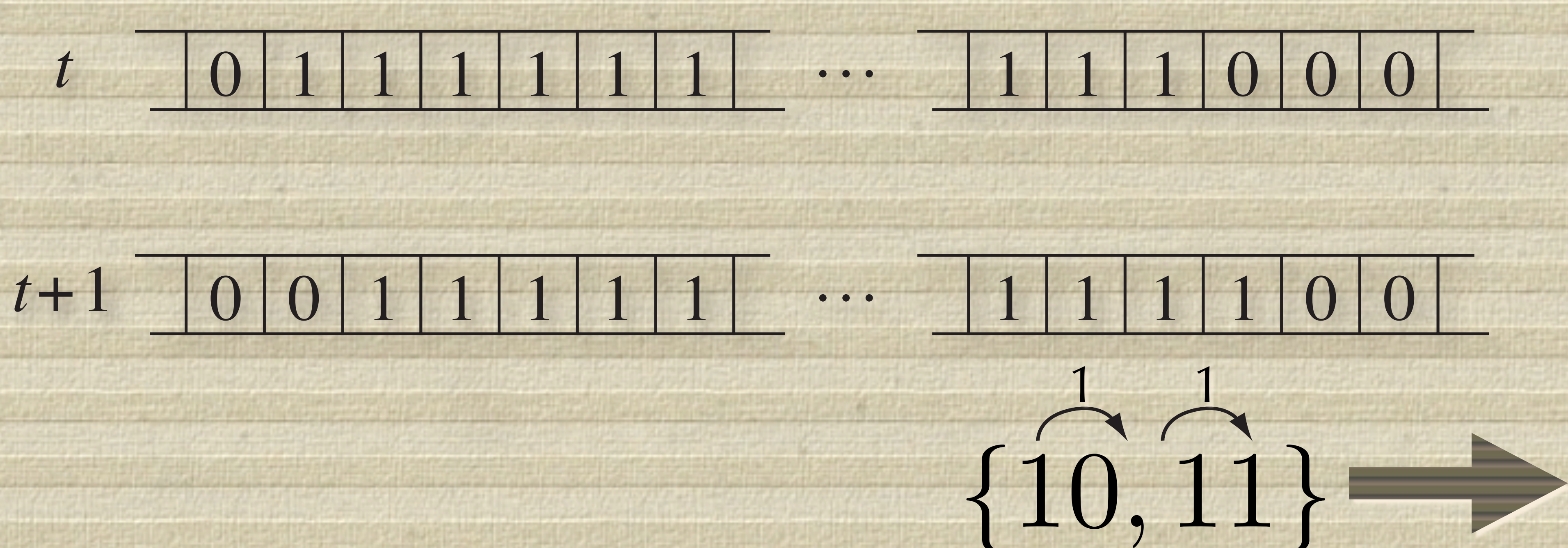


Universal Number-Conserving Cellular Automata

- 2002 Imai et, al. 26-state 2D, von Neumann
- 2003 Moreira: 1D, 2q+2-state, 2n-neighborhood NCCA can simulate q-state, n-neighborhood CA
- 2007: Tanimoto et, al.: 14-state, 2D, von Neumann

Motion representation

Any transition of number-conserving CA can be characterized by a set of movement of particles.



$$f(0,0,0)=0$$
$$f(0,0,1)=0$$
$$f(0,1,0)=0$$
$$f(0,1,1)=0$$
$$f(1,0,0)=1$$
$$f(1,0,1)=1$$
$$f(1,1,0)=1$$
$$f(1,1,1)=1$$

(cf. Fuks 2000, Pivato 2002)

2D case? (cf. Kari, Taati 2008)

Tomorrow

- How to play with cellular automata with various kind of neighborhoods in two and three-dimension, hyperbolic plane...

References

- Kari, J.: Theory of cellular automata: A survey, TCS 334 (2005) 3-33.
- Ollinger, N: Universalities in cellular automata: A short survey, JAC 2008, pp.102-118.