

## 数論セミナー

日時: 2026年5月11日(月) 17:00~

場所: D814 (+Teams)

講演者: Lama Tarsissi (Sorbonne University Abu Dhabi)

講演題目:

A geometric point of view on the synchronization of three Christoffel words

アブストラクト:

We study the collision-free superposition (“synchronization”) of Christoffel words of common length. Working on the Cayley cycle of  $\mathbb{Z}/n\mathbb{Z}$ , we introduce two matrices attached to a generator vector  $G = (g_1, \dots, g_\ell)$  and a shift vector (“seed”)  $V$ : the orbital matrix  $O(G, V)$ , whose entries are the orbit labels  $v_i + jg_i \pmod{n}$ , and the Christoffel-conjugate matrix  $C(G, V)$ , which marks wraparounds (“decreases”). A seed  $V$  synchronizes  $G$  iff each column of  $C(G, V)$  contains exactly one nonzero entry. We prove a short “vertical invariant”: the column-sum of  $O(G, V)$  is constant if and only if there is exactly one decrease between every pair of consecutive columns. Using this, we give a complete algebraic characterization of synchronizing seeds for  $\ell = 2$  and for  $\ell = 3$  when at least two generators coincide; we also cover a specific all-distinct family proportional to  $(1, 2, 4)$ . A geometric interpretation in terms of standard 4-connected Réveillé segments (with parameters directly read from  $V$ ) is presented in a final section.

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