New Aspects of Analytic Number Theory

October 27 (Mon.) – 29 (Wed.), 2008 at RIMS, Kyoto Univ.

Titles and Abstracts

Takao Komatsu (Hirosaki Univ.)

Four term leaping recurrence relations

Elsner and Komatsu investigated linear three-term recurrence formulae $Z_n = T(n)Z_{n-1}+U(n)Z_{n-2}$ $(n \geq 2)$ with sequences of integers $\{T(n)\}_{n\geq 0}$ and $\{U(n)\}_{n\geq 0}$, which are ultimately periodic modulo m. If $U(n) \neq 0$ for all $n \geq 0$, then there exists a three-term leaping recurrence relation of the form $W_1 Z_{rn+i} + W_2 Z_{r(n-1)+i} + W_3 Z_{r(n-2)+i} = 0$. In the simplest case $T(n) = U(n) = 1$ for all n, we know that $F_n - L_r F_{n-r} + (-1)^r F_{n-2r} = 0$, where F_n and L_n are Fibonacci and Lucas numbers, respectively. We shall observe four-term relations in various directions.

Vichian Laohakosol and Kantaphon Kuhapatanakul (Kasetsart Univ., Thailand)

Reciprocal sums of generalized second order recurrence sequences

A generalized second order recurrence sequence is a sequence whose elements satisfy a linear second order recurrence relation with non-constant coefficients. Several identities about reciprocal sums of elements in a generalized second order recurrence sequence are derived extending those of Hu, Sun and Liu in 2001. Applications to continued fractions are given.

Hajime Kaneko (Kyoto Univ.)

Applications of subspace theorem to the fractional parts of geometric series

A normal number in an integer base α is a positive number for which all finite words with letters from the alphabet $\{0, 1, \ldots, \alpha - 1\}$ occur with the proper frequency. Borel proved that almost all positive numbers are normal in all integer basis. He conjectured that any positive algebraic irrational number is normal in every integer base. However, we know no such number whose normality was proved.

Let λ_m be the number of nonzero digits among the first m digits of the α-ary expansions of algebraic irrational ξ. In case of $\alpha = 2$, Bailey, Borwein, Crandall, and Pomerance proved

$$\lambda_m > c(\xi) m^{1/D},$$

where $c(\xi)$ is a positive constant depending only on ξ and D is the degree of ξ. In case of $\alpha \geq 3$, Bugeaud showed

$$\lambda_m > (\log m)^{1+1/(w(\alpha)+4)} (\log \log m)^{-1/4},$$

where $w(\alpha)$ is the number of prime divisors of α.

The main purpose of this talk is to generalize these results to the fractional parts of the powers of algebraic numbers. We now recall the definition of Pisot and Salem numbers. Pisot numbers are algebraic integers greater than 1 whose conjugates different from themselves have absolute values strictly less than 1. Salem numbers are algebraic integers greater than 1 which have at least one conjugate with modulus 1 and exactly one conjugate outside the unit circle. Let $\alpha > 1$ be an algebraic number with minimal polynomial $a_d X^d + a_{d-1} X^{d-1} + \cdots + a_0 \in \mathbb{Z}[X]$. Set

$$L_+(\alpha) = \sum_{i=0}^{d} \max\{0, a_i\}, \quad L_-(\alpha) = \sum_{i=0}^{d} \max\{0, -a_i\}.$$

Take a positive number ξ. If α is a Pisot or Salem number, assume $\xi \notin \mathbb{Q}(\alpha)$. Dubickas verified for infinitely many natural number n that

$$\{\xi \alpha^n\} \geq \min\{1/L_+(\alpha), 1/L_-(\alpha)\}.$$

Note that in case of $\alpha \in \mathbb{N}$,

$$\{\xi \alpha^n\} \geq \min\{1/L_+(\alpha), 1/L_-(\alpha)\} = 1/\alpha$$

if and only if the $(n+1)$-th digit of α-ary expansion of ξ is nonzero. In this talk, we estimate the cardinality of the set

$$\lambda_m = \left\{ n \in \mathbb{N} \middle| 0 \leq n < m, \{\xi \alpha^n\} \geq \min\{1/L_+(\alpha), 1/L_-(\alpha)\} \right\}.$$

Takaaki Tanaka (Keio Univ.)

Algebraic independence of certain series involving continued fractions and generated by linear recurrences

Let $\Theta(x, a, q)$ be an analogue of a certain q-hypergeometric function generated by a linear recurrence $\{R_k\}$ whose typical example is the sequence $\{F_k\}$ of Fibonacci numbers. Main theorem gives the necessary and sufficient condition for the values $\Theta(x, a, q)$ at any distinct algebraic points (x, a, q) to be algebraically independent. Its corollaries give an analogue $E(x, q)$ of q-exponential function generated by $\{R_k\}$

and taking algebraically independent values for any distinct pairs (x, q) of nonzero algebraic numbers, and an irregular continued fraction $\Theta(a, q)$ generated by $\{R_k\}$ and taking algebraically independent values for any distinct pairs (a, q) of nonzero algebraic numbers.

Hideaki Ishikawa (Hachinohe National College of Technology) and Yuichi Kamiya (Meijigakuin Univ.)

On a relation between sums of arithmetical functions and Dirichlet series

We introduce the concept of sequences of good oscillating. The functions in sequences of good oscillating are slowly increasing, and are connected by a certain differential relation.

We prove that if certain error terms related to sums of arithmetical functions are of good oscillating, then Dirichlet series associated with those arithmetical functions can be continued analytically over the whole plane.

We also discuss the converse assertion that if Dirichlet series can be continued analytically over the whole plane and, additionally, a certain assumption holds, then error terms related to sums of Dirichlet coefficients are of good oscillating.

Hyunsuk Moon (Kyungpook National Univ., Korea)

On the structure of the Mordell-Weil groups of Jacobians over infinite number fields

Frey and Jarden have asked whether the Mordell-Weil group of every nonzero abelian variety defined over a number field K has infinite Mordell-Weil rank over the maximal abelian extension of K. Rosen and Wong proved the infiniteness of the rank for the Jacobian of any curve that can be realized over K as a cyclic geometrically irreducible cover of the projective line. In this talk, we will give another proof of Rosen-Wong's result together with slightly more precise information on the structure of the Mordell-Weil group.

Yoshinori Hamahata (Tokyo Univ. of Science)

Reciprocity laws of Dedekind sums in characteristic p

In 1989, S. Okada introduced Dedekind sums in function fields and established the reciprocity law for them. Inspired by his work, we introduced Dedekind sums in finite fields and established the reciprocity law for them. In our talk, we introduce Dedekind sums for lattices in function fields, finite fields, respectively. Our main result is the reciprocity law for them. It should be noted that our result generalizes the previous results.

Chris K. Caldwell (The University of Tennessee at Martin, USA)

Generalized Sierpiński Numbers

In 1960, W. Sierpiński proved that there are infinitely many odd integers k such that $k \cdot 2^n + 1$ is composite for all $n \geq 0$. These integers k are now called Sierpiński numbers. In 1962, J.L. Selfridge suggested that $k = 78557$ is the least Sierpiński, and the computing effort to prove Selfridge correct is still ongoing. We show that the are generalized Sierpiński numbers for every integer base $b > 1$ (integers k for which $k \cdot b^n + 1$ is always composite), and that for most integers $k > 1$, there is a base b for which k is a Generalized Sierpinski. We also address connections with the Mersenne and Fermat numbers, and briefly discuss the related computational issues.

This includes joint work with three of my undergraduate students: Amy Brunner, Daniel Krywaruczenko, and Chris Lownsdale.

Yoshiyuki Kitaoka (Meijo Univ.)

Distribution of roots of a polynomial modulo primes

Let $f(x)$ be a monic polynomial with integer coefficients. We observe a statistical relation of roots of $f(x)$ modulo p, where p runs over primes such that $f(x)$ decomposes completely modulo p. Based on it, we propose conjectures on the distribution of local roots of $f(x)$.

Takashi Nakamura (Kyushu Univ.)

Zeros and the universality for the Euler-Zagier-Hurwitz type of multiple zeta-functions

In this talk, we will show relations between the zero-free region and the universality for the Euler-Zagier-Hurwitz type of multiple zeta-functions. Roughly speaking these relations imply that we can obtain the universality for the Euler-Zagier-Hurwitz type of multiple zeta-functions by their zero-free property, and vice versa. Moreover, we obtain the non-trivial zeros, joint denseness and functional independence for the Euler-Zagier-Hurwitz type of multiple zeta-functions.

Makoto Minamide (Nagoya Univ.)

The number of zeros of the derivative of the modified Selberg zeta function

Luo studied the number of zeros of the derivative of the Selberg zeta function associated with a compact Riemann surface of genus $g \geq 2$ (Amer. J. Math. 127 (2005), 1141–1151). This investigation is motivated by the multiplicity problem for eigenvalues of the hyperbolic Laplacian.

In my talk we shall consider the case for the modular surface $PSL(2,\mathbb{Z})\backslash\mathbb{H}$. To this end, we define the modified Selberg zeta function $W(s)$ by

$$W(s) = \frac{Z(s)}{\zeta(2s)},$$

where $Z(s)$ is the Selberg zeta function for $PSL(2,\mathbb{Z})$ and $\zeta(s)$ is the Riemann zeta function. Let $N_1^v(T)$ be

$$N_1^v(T) = \sharp\{\beta' + i\gamma' | W'(\beta' + i\gamma') = 0, 1/2 \leq \beta', 0 < \gamma' \leq T\}.$$

One result is the following formula.

$$N_1^v(T) = \frac{1}{12}T^2 - \frac{2}{\pi}T\log T + O(T) \quad (T \to \infty).$$

Yasushi Komori (Nagoya Univ.), Kohji Matsumoto (Nagoya Univ.) and Hirofumi Tsumura (Tokyo Metropolitan Univ.)

Certain double series of Euler type and of Eisenstein type and Hurwitz numbers (I)

In this talk, based on the works of Hurwitz, Kronecker, Herglotz and Katayama, we consider certain generalized Eisenstein series. By introducing certain generalized Hurwitz numbers and their generating functions, we evaluate the generalized Eisenstein series in terms of them. Our present result includes our previous work about certain Dirichlet series involving hyperbolic functions, which can be regarded as a double analogue of the works of Cauchy, Mellin, Ramanujan, Berndt and so on.

Masatoshi Suzuki (Univ. of Tokyo)

On a continuous deformation of the Riemann zeta-function

In this talk, I introduce a deformation of the Rieamann zeta-function endowed with two continuous parameters. It has several nice symmetries. As a consequence of such symmetries, the distribution of zeros on the deformation is closely related to zeros of the Riemann zeta-function. This deformation contains the difference of two zeta-functions which was studied by J. C. Lagarias as a special case, and has a connection with de Branges' Hilbert space of entire functions.

Marc Huttner (Lille 1, France)

Constructible sets of linear differential equations and effective rational approximations of polylogarithmic functions

The aim of this article is to show how to investigate rational approximations to solutions of some linear differential equations from the perspective of moduli of linear differential equations with fixed monodromy group. One of the main arithmetic applications concerns the study of linear forms involving polylogarithmic functions.In particular,we give an explanation of Apery's and Gutnik's construction of simultaneous rational approximations of $\zeta(3)$ and of the well-poised hypergeometric origin of Rivoal's construction on linear forms involving odd Zeta values.

Michio Ozeki (Hirosaki Univ.)

A solution to a problem of S. Manni and the related topics

In a paper S. Manni posed several problems concerning identities between Siegel theta series of various degrees associated with even unimodular extremal lattices. In this talk the speaker gives an account of a solution to one of his problems. The talk also touches other problems and some related topics.

Shin-ichi Yasutomi (Suzuka National College of technology) and Jun-ichi Tamura (Tsuda College)

A new multidimensional continued fraction algorithm

It has been believed that the continued fraction expansion of (α, β) $(1, \alpha, \beta)$ is a $\mathbb{Q}-$basis of a real cubic field K) obtained by the modified Jacobi-Perron algorithm is periodic. We gave a numerical experiments from which we can guess the non periodicity of the expansion of $(\langle\sqrt[3]{3}\rangle, \langle\sqrt[3]{9}\rangle)$ ($\langle x\rangle$ denotes the fractional part of x). We give a new algorithm which is something like the modified Jacobi-Perron algorithm, and give some experiments by the new algorithm. By our experiments, we can expect that the expansion of (α, β) by our algorithm always becomes periodic for any real cubic field K.

Takumi Noda (Nihon Univ.)

An explicit formula for the zeros of the Rankin-Selberg L-function

We report one explicit formula for the zeros of the Rankin-Selberg L-function which proved by using the projection of the C^∞-automorphic forms. The projection was introduced by Sturm (1981) in the study of the special values of automorphic L-functions. Combining the idea of Zagier and the integral transformation of the confluent hypergeometric function, we obtain an explicit formula which correlates the zeros of the zeta-function and the Hecke eigenvalues.

Yasushi Komori (Nagoya Univ.), Kohji Matsumoto (Nagoya Univ.) and Hirofumi Tsumura (Tokyo Metropolitan Univ.)

Certain double series of Euler type and of Eisenstein type and Hurwitz numbers (II)

In this talk, we consider certain double series in two variables such as the Euler double zeta-function and its generalization of Eisenstein type. First we give some functional equations among these series and confluent hypergeometric functions. Next we show functional equations of traditional type for these double series which hold on certain hyperplanes. In particular, we give functional equations for the Euler double zeta-function. Furthermore we give some functional relations for these series and double series of another type involving hyperbolic functions. These results include certain value-relation formulas for these double series in terms of Hurwitz numbers.

Carsten Elsner (FHDW Hannover, Germany), Shun Shimomura (Keio Univ.) and Iekata Shiokawa (Keio Univ.)

Algebraic independence of values of Ramanujan q-series

Let $P(q)$, $Q(q)$, $R(q)$ be the Ramanujan functions

$$P(q) = 1 - 24 \sum_{n=1}^{\infty} \frac{nq^n}{1-q^n}, \qquad Q(q) = 1 + 240 \sum_{n=1}^{\infty} \frac{n^3 q^n}{1-q^n},$$
$$R(q) = 1 - 504 \sum_{n=1}^{\infty} \frac{n^5 q^n}{1-q^n}.$$

Nesterenko proved that, for $q \in \overline{\mathbb{Q}}$ with $0 < |q| < 1$, the values $P(q)$, $Q(q)$, $R(q)$ are algebraically independent. Consider the q-series

$$A_{2j+1} = A_{2j+1}(q) = \sum_{n=1}^{\infty} \frac{n^{2j+1} q^{2n}}{1-q^{2n}} \quad (j = 0, 1, 2, \ldots) \tag{1}$$

for $|q| < 1$. The first three $A_1(q^{1/2})$, $A_3(q^{1/2})$, $A_5(q^{1/2})$ define the Ramanujan functions. We study the algebraic independence of the values $A_{2j+1}(q)$ for $j \geq 0$. Our result is stated as follows:

Theorem 1. *If $q \in \overline{\mathbb{Q}}$ with $0 < |q| < 1$, then, for any distinct positive integers j_1, j_2 with $(j_1, j_2) \neq (1, 3)$, the numbers $A_1(q)$, $A_{2j_1+1}(q)$, $A_{2j_2+1}(q)$ are algebraically independent. Furthermore, as q-series, $A_3(q)$ and $A_7(q)$ satisfy the relation*

$$A_7(q) = A_3(q) + 120 A_3(q)^2.$$

Our method of proving this theorem can be applied to another q-series, e.g.

$$L_{2j+1} = L_{2j+1}(q) = \sum_{n=1}^{\infty} \frac{n^{2j+1} q^{2n}}{1+q^{2n}} \quad (j = 0, 1, 2, ...). \tag{2}$$