On sums of cubes of smooth numbers.

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In 1997, Harcos considered Waring's problem for smooth numbers, that is, problems concerning representations of numbers as sums of powers of natural numbers that have smaller prime factors only. In particular, he showed that for each $s \geq 9$, every sufficiently large integer n can be written as the sum of s cubes of natural numbers that have no prime factor exceeding $\exp(c(\log n \log\log n)^{1/2})$ with some positive constant c. Further, Brüdern and Wooley succeeded in establishing the corresponding result for $s = 8$.

As regards the case $s = 7$, the following result had just recently obtained as a by-product of my research on sums of seven cubes of almost primes: every sufficiently large n can be written as a sum of seven cubes of natural numbers free of prime factors exceeding $n^{\tau/3}$, provided that $\tau > (-2784931 + 53672\sqrt{2833})/85977$ $(= 0.835\cdots)$. (Here note that this statement is trivial if $\tau = 1$, in view of Linnik's seven cube theorem.) This conclusion, though appears weak, was the first non-trivial result for the sum of seven cubes of smooth numbers. In this talk, a further minor improvement in this direction will be reported.