Simultaneous Pell Equation of Indefinite Signature

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Let a_1, a_2, b_1 and b_2 be given positive integers such that a_1a_2, b_1b_2 and $a_1a_2b_1b_2$ are non-square. We consider the system

$$\begin{cases} a_1 x^2 - a_2 z^2 &= \pm 4, \\ b_1 y^2 - b_2 z^2 &= \pm 4 \end{cases}$$

of Diophantine equations in unknown positive integers x, y and z, where signature of the right hand side may depend on x, y and z. We show this equation has at most 3 solutions under the technical assumption $\max\{a_1, a_2, b_1, b_2\} \geq 10^{15}$.