DIOPHANTINE APPROXIMATION IN POSITIVE CHARACTERISTIC

Formal power series over a given field have been studied for a long time in Number Theory. The case of a finite base field is particularly important and the analogy between these power series and the real numbers is striking. Nevertheless the positive characteristic, inducing the existence of the Frobenius isomorphism, makes rational approximation to algebraic elements very different from the case of real numbers and somehow more complex. Here we give a short account of different steps in the study of diophantine approximation in the function field case (see below in chronological order [1]-[10] and also [11] for more references). In connection with this topic, we also describe three families of sequences in a finite field derived from an algebraic continued fraction.

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