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Diophantine analysis and words
by

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Abstract

The *Fibonacci word* on an alphabet $\{a, b\}$ with two letters is the infinite word

$$w = abaababaabaababaababaabaababaabaab\dots$$

which is the fixed point of the morphism $a \mapsto ab$, $b \mapsto a$, This word is the limit of the sequence $(f_n)_{n\ge1}$ of finite words, starting with $f_1 = b$, $f_2 = a$, which is defined by the induction formula $f_n = f_{n-1}f_{n-2}$ for $n \ge 3$. The word w is one of the simplest *Sturmian words*, namely a non-periodic word of minimal complexity, as the number $p(m)$ of factors of length m of w is $m + 1$ for any $m \ge 1$.

If one replaces a and b by two distinct positive integer, then the real number ξ, whose continued fraction expansion is obtained from w by this process, is a transcendental number. Recently, D. Roy found that this number ξ has peculiar properties as far as the simultaneous approximation of ξ and ξ^2 by rational numbers is concerned. Moreover, further recent results on this topic are due to D. Roy, M. Laurent, Y. Bugeaud and others.

Instead of taking continued fraction expansions, one can fix a basis $g \ge 2$ and then associate to a word on an alphabet with g letters the number x whose g-ary expansion is obtained by replacing the letters of the alphabet by $0, 1, \dots, g-1$. Again, if we start from the Fibonacci number, or more generally from a Sturmian word, the corresponding number x is transcendental. B. Adamczewski and Y. Bugeaud recently proved that, for algebraic irrational number x, the complexity of the associated word w satisfies

$$\liminf_{n\to\infty} \frac{p(n)}{n} = +\infty.$$

They deduce that the g-ary expansion of an algebraic irrational number cannot be generated by a finite automaton.

A further connection between words and Diophantine analysis arises in the study of multiple zeta values.

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