We consider the divergence behavior of certain infinite matrix products and continued fractions in terms of limit sets. The limit set of a sequence is defined to be the set of limits of convergent subsequences. We find that for a large naturally defined class of sequences constructed from infinite matrix products, these sets are extremely well behaved. In one specific instance, we compute them explicitly and show how a mysterious result of Ramanujan fits into our general theory. Other results include a theorem which vastly extends the classic Stern-Stolz theorem on divergent continued fractions. We also mention other applications to Poincare type recurrences and matrix continued fractions.