# Diophantine Analysis and Related Fields 2020
# Scheduled Talks and Their Abstracts

## Thursday 5th March

13:40–14:30 **Yasutsugu Fujita** (Nihon University)

Title: **Generators and integral points on quartic curves of the forms** $u^2 \pm v^4 = m$

Abstract: In the preceding work, we investigated generators and integral points on twists of Fermat's cubic of the form $x^3 + y^3 = m$ for a non-zero integer $m$. In this talk, using a similar strategy we study generators and integral points on quartic curves of the forms $u^2 \pm v^4 = m$ for a non-zero integer $m$. The theorems assert that certain integral points on the curves can be extended to bases for the Mordell-Weil groups of the elliptic curves birationally equivalent to the quartic curves in the cases where the Mordell-Weil ranks are at most two. Moreover, the integral points on the quartic curves are explicitly described in the rank one or two case.

14:40–15:30 **Alan Filipin** (University of Zagreb)

Title: **The extension of parametric Diophantine triples in the ring of Gaussian integer**

Abstract: A Diophantine $m$-tuple is a set of $m$ distinct integers such that the product of any two of its distinct elements increased by 1 is a perfect square. In this talk we consider the extendibility of a parametric Diophantine triple $\{k-1, k+1, 16k^3-4k\}$ in the ring of Gaussian integers to a Diophantine quadruple. Similar parametric family $\{k-1, k+1, 4k\}$ was studied by Franušić and it was proven that the extension to a Diophantine quadruple is unique.

The family of the form $\{k-1, k+1, 16k^3-4k\}$ was studied in rational integers by Bugeaud, Dujella and Mignotte and it was a special case while solving the extendibility problem of Diophantine pairs of the form $\{k-1, k+1\}$, in which it was not possible to use the same method as in the other cases. As authors pointed out, the difficulty appears because the gap between elements $k+1$ and $16k^3-4k$ is not sufficiently large. We find the same difficulty here while trying to use Diophantine approximations. However, we partially solve this problem by using linear forms in logarithms.

During the proof we have also proven some results on the extendibility of general triple $\{a, b, c\}$ in Gaussian integers which can be useful in improving the bound for the size of Diophantine $m$-tuples in Gaussian integers.

This is joint work with Nikola Adžaga and Zrinka Franušić.

15:50–16:40 **Shinichi Yasutomi** (Toho University)

(Joint work with Rao Hui (Central China Normal University) and Jiang Zhu (Huazhong University of Science and Technology))

Title: **Ergodicity for $p$-adic multidimensional continued fraction algorithms**

Abstract: Schweiger provided a generalization of multidimensional continued fraction algorithms as piecewise fractional linear maps. In this talk following Schweiger's generalization of multidimensional continued fraction algorithms we consider a family of $p$-adic multidimensional continued fraction algorithms, which include Schneider's algorithm and Ruban's algorithms. We give the ergodicity of transformations associated with algorithms which is included in the family. We also give some numerical experiments related to the topic. This is a joint work with Rao Hui and Jiang Zhu.

16:50–17:40 **Kota Saito** (Nagoya University)

Title: **Distribution of arithmetic progressions of Piatetski-Shapiro sequences**

Abstract: For every $r \geq 1$ and $1 < \alpha < 2$, we get the asymptotic formula of the distribution of positive integers $n$ such the set of integer parts of $n^\alpha, (n+r)^\alpha, \cdots, (n+r(k-1))^\alpha$ is an arithmetic progression of length $k$. The asymptotic density of this set is equal to $1/(k-1)$ that is independent of $r \geq 1$ and $1 < \alpha < 2$.

## Friday 6th March

10:00–10:50 **Hajime Kaneko** (Tsukuba University)

Title: **Application of Hensel's lemma for the base-b expansions of integers**

Abstract: Let $b \geq 2$ be a fixed integer. The base-$b$ expansions of special integers have been investigated by many mathematicians. For instance, Dupuy and Weirich conjectured the uniformity of the digits in the ternary expansions of $2^n$ $(n = 0, 1, 2, \ldots)$. For investigating the digits of base-$b$ expansions of integers, we generalize Hensel's lemma, which gives an answer for an open problem by Axelsson and Khrennikov. This is a joint work with Thomas Stoll.

11:10–12:00 **István Pink** (University of Debrecen)

Title: **Some applications of Baker's method to Diophantine equations**

Abstract: In this talk we will concentrate on some applications of the famous method of Baker to Diophantine equations. After a concise introduction to the theory of linear forms in logarithms of algebraic numbers we give a brief survey on some classical results in Diophantine number theory involving Baker's method. Then we point out several refinements of the original work of Baker and we indicate how the combination of these refinements with some other effective methods (e.g. LLL algorithm, local method, hypergeometric method, etc.) may lead to effective resolution of Diophantine equations of certain type. We end the talk with explicit resolution of some concrete Diophantine equations illustrating the power of the method.

14:00–14:50 **Seiji Nishioka** (Yamagata University)

Title: **Applications of difference algebra to Mahler functions**

Abstract: Proofs of the transcendence or algebraic independence of values of Mahler functions require that of the functions themselves. The algebraic relation of Mahler function $f(x)$ and its transforms $f(x^d)$, $f(x^{d^2})$, etc. can be regarded as an difference equation by the operator transforming $y(x)$ to $y(x^d)$ in difference algebra which is a field considering algebraic properties of solutions. Hence results in difference algebra would be applicable. In this talk, some examples will be introduced.

15:00–15:50 **Yu Yasufuku** (Nihon University)

Title: **Towards Vojta's conjecture on blowups of $\mathbb{P}^2$**

Abstract: Vojta's conjecture is a deep conjecture in Diophantine geometry, quantitatively describing how the Diophantine approximation on a variety is controlled by the geometry. In the case of blowups of $\mathbb{P}^2$, this conjecture becomes a GCD inequality, and this inequality has been proved for some subsets (Corvaja–Zannier, Luca). In this talk, we discuss a weaker version of the conjectured inequality for the full set.

16:10–17:00 **Makoto Kawashima** (Osaka University)

Title: **Rodligues' formula and irrationality**

Abstract: The Padé approximants of logarithm function are given by A. M. Legendre in 1782. Nowadays they are called as Legendre polynomials. After the work of Legendre, in 1816, O. Rodligues found a simple representation of Legendre polynomials which is known as Rodligues' formula. In this talk, we will show a generalization of Rodligues' formula and give explicit construction of Padé approximants of certain class of formal Laurent series including Lerch functions. We also give some applications of the Padé approximants, for example, irrationality and linear independence of values of Lerch functions and some other related functions.

17:10–18:00 **Tomohiro Yamada** (Osaka University)

Title: **Irrationality measure of** $(\arctan 1/2)/\pi$

Abstract: We shall show that for all positive integers $p, q$ with $q$ sufficiently large,

$$|p\pi/4 - q \arctan 1/2| > q^{-9.04722}$$

## Saturday 7th March

9:30–10:20 **Masanori Katsurada** (Keio University)

Title: **Complete asymptotic expansions for the product average of higher derivatives of Lerch zeta-functions**

Abstract: Let $s = \sigma + it$ and $s_j = \sigma_j + it_j$ ($\sigma, \sigma_j, t, t_j \in \mathbb{R}$) for $j = 1, 2$ be complex variables, $x$ and $\lambda$ real parameters with $x > 0$, and write $e(s) = e^{2\pi is}$. The Lerch zeta-function $\phi(s, x, \lambda)$ is defined by the Dirichlet series $\sum_{l=0}^{\infty} e(\lambda l)(x + l)^{-s}$ ($\sigma > 1$), and its meromorphic continuation over the whole $s$-plane. We write $\phi^{(m)}(s, x, \lambda) = (\partial/\partial s)^m \phi(s, x, \lambda)$ ($m = 0, 1, \ldots$), and introduce the product average of $\phi^{(m)}(s, x, \lambda)$, in the form

$$\mathcal{I}_{m_1,m_2}(s_1, s_2; a, \lambda) = \int_0^1 \phi^{(m_1)}(s_1, a + x, \lambda)\phi^{(m_2)}(s_2, a + x, -\lambda)dx$$

for any nonnegative integers $m_1$ and $m_2$, and any real $a$ and $\lambda$ with $a > 0$. It is then shown in the talk that complete asymptotic expansions exist for $\mathcal{I}_{m_1,m_2}(\sigma_1 + it, \sigma_2 - it; a, \lambda)$ in the descending order of $t$ as $t \to \pm\infty$, being valid in a satisfactorily wide region of $(\sigma_1, \sigma_2)$ and for any nonnegative integers $m_1$ and $m_2$. The case $(\sigma_1, \sigma_2) = (\sigma, \sigma)$ and $(m_1, m_2) = (m, m)$ of our main result in particular reduces to complete asymptotic expansions for the mean square $\int_0^1 |\phi^{(m)}(\sigma + it, a + x, \lambda)|^2 dx$ ($m = 0, 1, \ldots$) as $t \to \pm\infty$, except the cases $\sigma = n/2$ ($n = 2, 1, 0, -1, \ldots$), where the important situations on the critical line $\sigma = 1/2$ and on the vertical line $\sigma = 1$ are treated as their limiting forms when $\sigma \to 1/2$ and $\sigma \to 1$, respectively.

Crucial rôles in the proofs are played by the complete asymptotic expansions (together with exact reminders) for a certain quotient of gamma functions.

10:40–11:30 **Takuya Aoki** (Waseda University)

Title: **On class number calculations of the intermediate fields of the cyclotomic $\mathbb{Z}_2$-extension over $\mathbb{Q}(\sqrt{5})$**

Abstract: Let $h_{2,n}$ be the class number of $\mathbb{Q}(2\cos(2\pi/2^{n+2}))$. It is known as Weber's class number problem whether $h_{2,n} = 1$ holds for all positive integer $n$ or not. This problem has been studied deeply by K. Horie, K. Horie-M. Horie, Fukuda-Komatsu, Okazaki, Morisawa-Okazaki and J. Miller.

In this talk, we attempt to generalize this problem for the case of $\mathbb{Q}(\sqrt{5})$ and introduce the results we obtained as analogies of the known results for the original Weber's class number problem.

11:40–12:30 **Ryotaro Okazaki**

Title: **On total capitulation in CM-fields and Scholz reflection theorem**

Abstract: We use A.Scholz's construction of class field to prove:

*Total Capitulation Thereom in CM-fields:*

*Let $K$ be a given CM-field. Then, there exists some extension field $L$ of $K$, so that*

- *The extension field $L$ is also a CM-field.*
- *All ideal classes of the subfield $K$ capitulate in the extension field $L$.*

This result is surprising since Horie's Divisibility Theorem asserts the relative ideal class number of $K$ divides that of $L$; and Fujii's Stability Theorem (2019) asserts the relative ideal class group of $K$ almost survives in $L$ if $K$ and $L$ are cyclotomic fields.