Diophantine Analysis and Related Fields 2015
Scheduled Talks and Their Abstracts

Thursday 5 March

13:40–14:30
Takafumi Miyazaki (Nihon University)
Title: *A polynomial-exponential Diophantine equation related to the sum of consecutive k-th powers.*
Abstract: Lucas's square pyramid problem asks to solve the Diophantine equation

$$1 + 2^2 + 3^2 + \cdots + x^2 = y^2.$$

It is known that the only non-trivial positive solution is $(x, y) = (24, 70)$, that is,

$$1 + 2^2 + 3^2 + \cdots + 24^2 = 70^2.$$

Here, we consider the following general equation:

$$1 + 2^k + 3^k + \cdots + x^k = y^n$$

in integers k, n, x, y with $k \geq 1,\ n \geq 2,\ x \geq 2,\ y \geq 2$. If the pair (k, n) is one of

$$(1, 2),\ (3, 2),\ (3, 4),\ (5, 2),$$

then the problem is reduced to solve a Pellian equation, in particular, we find infinitely many solutions (x, y). In 1956, Schäffer conjectured that the answer to Lucas's problem mentioned above is the only solution of our equation for which the pair (k, n) does not belong to $\{(1, 2),\ (3, 2),\ (3, 4),\ (5, 2)\}$. This is still unsolved in spite of many studies in the literature. In this talk, we solve our equation in a special case, and our result gives an affirmative answer to Schäffer's problem. This is a joint work with A. Bérczes, L. Hajdu and I. Pink (University of Debrecen).

14:40–15:20
Tomohiro Ooto (Tsukuba University)
Title: *Quadratic approximation of continued fractions with low complexity in* $\mathbb{F}_q((T^{-1}))$
Abstract: We consider the Koksma's exponent w_2^* in $\mathbb{F}_q((T^{-1}))$. We obtain a finiteness of the value of continued fractions with low complexity and some conditions taken by the function w_2^*. In particular, automatic or primitive morphic continued fractions satisfy the conditions.

15:40–16:30
Takumi Noda (Nihon University)
Title: *Confluent hypergeometric type Ramanujan's formulas*
Abstract: Let $\zeta(s, x)$ be the Hurwitz zeta-function. Ramanujan's formula (1916-17) shows that $\zeta(s, x)$ is one generating function of the Riemann zeta-function, (the binomial type power series):

$$\zeta(s, 1 + x) = \sum_{m=0}^{\infty} \frac{\Gamma(s + m)}{\Gamma(s)\, m!} \zeta(s + m)(-x)^m,$$

for $|x| < 1$ and $s \in \mathbb{C} \backslash \{-1, 0, 1, 2, \cdots\}$. In this talk, we introduce some new zeta-functions associated with confluent hypergeometric functions as classical Dirichlet series. We will show functional properties of these zeta-functions which lead to analogues of Ramanujan's formula.

16:40–17:30

Yasutsugu FUJITA (Nihon University)

Title: *Bounds for Diophantine quintuples*

Abstract: A set $\{a_1, \dots, a_m\}$ of m positive integers is called a Diophantine m-tuple if $a_i a_j + 1$ is a perfect square for all i, j with $i \neq j$. A folklore conjecture states that there exists no Diophantine quintuple. It has been known that various kinds of Diophantine triples or pairs cannot be extended to Diophantine quintuples, such as $\{1, 3, 8\}$ by Baker and Davenport or more generally $\{k-1, k+1\}$ with $k \geq 2$ an integer. In this talk we will give some recent results on the Diophantine pairs $\{a, b\}$ with $a < b$ that cannot be contained in Diophantine quintuples $\{a, b, c, d, e\}$ with $b < c < d < e$, one of which is the pair $\{a, b\}$ with $b \leq 3a$ (joint work with Mihai Cipu).

Friday 6 March

9:30–10:10

Jonathan CAALIM (Tsukuba University)

Title: *Rotational beta expansion: Ergodicity and Soficness*

Abstract: Let $1 < \beta \in \mathbb{R}$ and $\zeta \in \mathbb{C} \setminus \mathbb{R}$ with $|\zeta| = 1$. Fix $\xi, \eta_1, \eta_2 \in \mathbb{C}$ with $\eta_1/\eta_2 \notin \mathbb{R}$. Then $\mathcal{X} = \{\xi + x\eta_1 + y\eta_2 \mid x \in [0,1), y \in [0,1)\}$ is a fundamental domain of the lattice $\mathcal{L}$ generated by η_1 and η_2 in $\mathbb{C}$. We define a map $T : \mathcal{X} \to \mathcal{X}$ by $T(z) = \beta\zeta z - d$ where $d = d(z)$ is the unique element in $\mathcal{L}$ such that $\beta\zeta z \in \mathcal{X} + d$. The above map gives a two-dimensional generalization of the positive and negative β-transformations. In this talk, we discuss the ergodicity and soficness of T. This is a joint work with Prof. Shigeki Akiyama.

10:20–11:10

Yu YASUFUKU(Nihon University)

Title: *Logarithmic Kodaira dimension and Diophantine equations*

Abstract: We describe a relationship between integral points on surfaces and certain Diophantine equations. On one hand, we have the structure theorems based on log kodaira dimension, studied by many Japanese mathematicians from the 70's. On the other hand, we have Diophantine tools, such as unit equations and Roth's theorem. We use both of these machineries to study Diophantine equations through log kodaira dimension. In particular, we obtain what we believe to be new results on finiteness of solutions to certain Diophantine equations. If time permits, we will also discuss an application to arithmetic dynamics in dimension 2. This is a joint work with Aaron Levin (Michigan State).

11:20–12:10

Masanori KATSURADA (Keio Univ.)

Title: *Asymptotic expansions for the Laplace-Mellin and Riemann-Liouville transforms of Lerch zeta-functions*

Abstract: The Laplace-Mellin and Riemann-Liouville type transforms are first introduced in a generic manner under certain natural settings. It is then shown that complete asymptotic expansions exist for these transforms applied to Lerch zeta-functions, when the pivotal parameter z in the transformations tends to both 0 and ∞ through the sector $|\arg z| \leq \pi/2$. Several applications of these results are also to be given.

14:00–14:50
Hajime KANEKO(Tsukuba University)
Title: *On the beta-expansions of algebraic numbers by a Pisot or Salem number beta*
Abstract: Let $\beta > 1$ be a real number. Rényi introduced the notion of β-expansions of real numbers which is applicable to number theory and dynamical systems. In particular, it is important to study the digits in the β-expansions of algebraic numbers in the case where β is a Pisot or Salem number. In this talk, we give new lower bounds for the number of nonzero digits.

15:00–15:40
Junya IWAKI (Gunma Univ.)
Title: *Linear independence measures for the values of certain Mahler functions*
Abstract: Let $r \geq 2$ and $m \geq 1$ be integers. Assume that an m-dimensional vector $\mathbf{F}(z) = {}^t(f_1(z), \cdots, f_m(z)) \in \mathbb{Q}[[z]]^m$ is a solution of a system of Mahler functional equations $\mathbf{F}(z^r) = \mathbf{A}(z)\mathbf{F}(z)$ with $\mathbf{A}(z) \in GL(m, \mathbb{Q}(z))$ such that $f_1(z), \cdots, f_m(z)$ converge on $|z| < R$ with $R > 0$ and that $1, f_1(z), \cdots, f_m(z)$ are linearly independent over $\mathbb{Q}(z)$.

In this talk, we give an upper bound of the linear independence measure for the numbers $1, f_1(1/b), \cdots, f_m(1/b)$, where $b > 1$ is an integer with $1/b < R$ such that b^{-r^k} are neither zeros nor poles of $\det \mathbf{A}(z)$ for all integers $k \geq 0$. For example, let $\mathbf{R}(x)$ and $\mathbf{B}(x)$ be the generating function of the Rudin-Shapiro sequence and that of Baum-Sweet sequence, respectively. Then we see that the linear independence measure of $1, \mathbf{R}(1/b), (-1/b)$ is at most 16, and that of $1, \mathbf{B}(1/b), \mathbf{B}(1/b^2)$ is also at most 16. This is a joint work with Prof. Masaaki Amou.

16:00–16:40
Akinari GOTO (Keio University), Taka-aki TANAKA (Keio University)
Title: *Algebraic independence of the values of Mahler functions in a certain case of positive characteristic*
Abstract: Mahler's method gives algebraic independence of the values of functions satisfying functional equations such as (1) below, which has recently been studied over base field $K := \mathbb{F}_q(\theta)$ of characteristic $p > 0$. For example, Denis treated every period of Carlitz's exponential as a Mahler's value and Pellarin gave, using Mahler's method, completely different proofs for the results of Papanikolas and of Chang and Yu for the values of its inverse function, Carlitz's logarithm. We consider different type of functional equations in positive characteristic than those treated in the previous works mentioned above. For an integer $d \geq 2$, define

$$f(x, z) := \sum_{k=0}^{\infty} x^k z^{d^k},$$

which satisfies the functional equation

$$f(x, z) = x f(x, z^d) + z. \tag{1}$$

The main result of this talk is the following:

Theorem. *Assume that the characteristic p does not divide d and that nonzero elements $a_1, \ldots, a_m$ of $K^{\text{alg.}}$, the algebraic closure of K, are distinct. Then for any $\alpha \in K^{\text{alg.}}$ with $0 < |\alpha|_v < 1$, the values*

$$\frac{\partial^j}{\partial x^j} f(a_i, \alpha) \quad (1 \leq i \leq m,\ 0 \leq j \leq p-1)$$

are algebraically independent over K. Hence, for the function $F(x) := \sum_{k=0}^{\infty} \alpha^{d^k} x^k$, the infinite set of the values

$$\left\{ F^{(j)}(a) \;\middle|\; 0 \leq j \leq p-1,\ a \in K^{\mathrm{alg.}} \backslash \{0\} \right\}$$

is algebraically independent over K.

16:50-17:40

Makoto KAWASHIMA (Osaka University)

Title: *Linear independence of values of certain p-adic functions*

Abstract: In this talk, we study two types of criteria of linear independence of p-adic numbers. First, we introduce a criterion of linear independence of p-adic numbers due to Pierre Bel. Then, by using the above result, we axiomatize two types of criteria of linear independence of special values of certain p-adic functions. Finally, we show the linear independence of special values of p-adic Lerch function by using this axiom.

Saturday 7 March

9:30-10:20

Hirofumi NAGOSHI (Gunma University)

Title: *The existence of zeros of non-primitive L-functions for $SL(2,\mathbb{Z})$ in the strip* $1/2 < \Re s < 1$

Abstract: Let $f(z)$ be a holomorphic cusp form of even positive integral weight for $SL(2,\mathbb{Z})$. In this talk, we will show that if $f(z)$ is not a Hecke eigenform then the associated L-function $L(s,f)$, which satisfies a functional equation, has infinitely many zeros in the strip $1/2 < \Re s < 1$, where the critical line of $L(s,f)$ is normalized to be $\Re s = 1/2$. Actually, a stronger assertion holds. In the proof, Kronecker's approximation theorem in the theory of Diophantine analysis and other things are used. A similar result was obtained by Voronin for certain Epstein zeta functions. However, we cannot adapt his proof straightforward to our case. Another similar result was obtained by Kaczorowski and Kulas for certain L-functions of degree 1 in the extended Selberg class, from a variant of the so-called "joint universality" property of Dirichlet L-functions. It should be noted that our proof as well as Voronin's proof does not use such an universality property.

We would also like to discuss some results concerning the independence of L-functions $L(s,f_j)$ for distinct primitive forms $f_j(z)$, since these results are closely related to the above one.

10:40-11:30

Takao KOMATSU (Wuhan university)

Title: *Associate and restricted Stirling numbers and their applications*

Abstract: We define the associate Stirling numbers of the first kind and of the second kind, and the restricted Stirling numbers of the first kind and of the second kind, as generalizations of the classical Stirling numbers of the first kind and of the second kind. By studying their arithmetical and combinatorial properties, we give several their applications.