The Fourth China-Japan Conference on Number Theory

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Shigeki Akiyama, Niigata University, Japan
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Wenpeng Zhang, Northwest University, China

Short Communications:

Kentaro Ihara, Kinki University, Japan
Takako Kuzumaki Kobayashi, Gifu University, Japan
Hailong Li, Weinan Teacher's College, China
Huaning Liu, Northwest University, China
Guangshi Lü, Shandong University, China
Masayuki Toda, Kinki University, Japan
Deyu Zhang, Shandong University, China

Time: August 30 - September 3, 2006

Venue: Shandong University Academic Center (Weihai)

Supported by JSPS, NSFC, and Shandong University

Scientific Program

Wednesday, August 30, 2006

Opening Address
The Organizers

Time: 9:00-9:40

Morning Session
Chair: Jianya Liu

Time: 10:00-10:40

Speaker: Andrzej Schinzel
The Number of Solutions in a Box of a Linear Homogeneous Congruence

Time: 11:00-11:40

Speaker: Shou-Wu Zhang
Periods Integrals and Special Values of L-series

Lunch: 12:00 Shandong University Academic Center

Afternoon Session
Chair: Krishnaswami Alladi

Time: 14:30-15:00

Speaker: Zhi-Wei Sun
Curious Identities and Congruences Involving Bernoulli Polynomials

Time: 15:20-15:40

Speaker: Shigeki Akiyama
Rational Based Number System and Mahler's Problem

Time: 15:50-16:10

Speaker: YoungJu Choie
Broué-Enguehard Maps and Atkin-Lehner Involutions

Time: 16:20-16:40

Speaker: Xiumin Ren

Estimates of Exponential Sums over Primes and Applications in Number Theory

Time: 16:50-17:05

Speaker: Haruo Tsukada
A General Modular Relation Associated with the Riemann Zeta-function

Time: 17:10-17:25

Speaker: Kenji Nagasaka
Density, Probability and Applications

Time: 17:30-17:40

Speaker: Takako Kuzumaki Kobayashi
A Transformation Formula for Certain Lambert Series

Dinner: 18:30 Shandong University Academic Center

Thursday, August 31, 2006

Morning Session
Chair: Vladimir N. Chubarikov

Time: 9:00-9:40

Speaker: Krishnaswami Alladi
New Approaches to Jacobi's Triple Product Identity and a Quadruple Product Extension

Time: 10:00-10:40

Speaker: Trevor D. Wooley
Waring's Problem in Function Fields

Time: 11:00-11:40

Speaker: Yangbo Ye
A New Bound for Rankin-Selberg L-functions

Lunch: 12:00 Shandong University Academic Center

Afternoon Session
Chair: Yoshio Tanigawa

Time: 14:30-15:00

Speaker: Wenpeng Zhang
The Mean Value of Dedekind Sum and Cochrane Sum in Short Intervals

Time: 15:20-15:50

Speaker: Isao Wakabayashi
Some Thue Equations and Continued Fractions

Time: 16:10-16:30

Speaker: Yuk-Kam Lau
The Error Terms in Dirichlet's Divisor Problem, the Circle Problem and the Mean Square Formula of the Riemann Zeta-function

Time: 16:40-17:00

Speaker: Wenguang Zhai
On the Fourth Power Moments of $\Delta(x)$ and $E(t)$

Time: 17:10-17:20

Speaker: Tianxin Cai / Xia Zhou
On the Height of Happy Numbers

Time: 17:25-17:35

Speaker: Masayuki Toda
On Gauss' formula for ψ and finite expressions for the L-series at 1

Time: 17:40-17:50

Speaker: Kentaro Ihara
On the Structure of Algebra of Multiple Zeta Values

Dinner: 18:30 Shandong University Academic Center

Friday, September 1, 2006

Excursion and Shopping (TBA)

Saturday, September 2, 2006

Morning Session
Chair: Kohji Matsumoto

Time: 9:00-9:40

Speaker: Vladimir N. Chubarikov
Trigonometric Sums in Number Theory, Analysis and Probability Theory

Time: 10:00-10:40

Speaker: Winfried Kohnen
Sign Changes of Fourier Coefficients and Hecke Eigenvalues of Cusp Forms

Time: 11:00-11:40

Speaker: Igor Shparlinski
Exponential and Character Sums with Combinatorial Sequences

Lunch: 12:00 Shandong University Academic Center

Afternoon Session
Chair: Wenpeng Zhang

Time: 14:30-15:00

Speaker: Chaohua Jia
On a Conjecture of Yiming Long

Time: 15:20-15:40

Speaker: Koichi Kawada
On the Sum of Five Cubes of Primes

Time: 15:50-16:10

Speaker: Honggang Xia
On Zeros of Cubic L-functions

Time: 16:20-16:40

Speaker: Zhiguo Liu
A Theta Function Identity to the Quintic Base

Time: 16:50-17:05

Speaker: Masaki Sudo
On the Exponential Equations $a^x - b^y = c\,(1 \leq c \leq 300)$

Time: 17:25-17:40

Speaker: Yoshinobu Nakai
TBA

Time: 17:45-17:55
Speaker: Deyu Zhang
Zero Density Estimates for Automorphic L-functions

Dinner: 18:30 Shandong University Academic Center

Sunday, September 3, 2006

Morning Session
Chair: Winfried Kohnen

Time: 9:00-9:40

Speaker: Daqing Wan
L-functions of Infinite Symmetric Powers

Time: 10:00-10:30

Speaker: Yoshiyuki Kitaoka
Distribution of Units of an Algebraic Number Field

Time: 10:50-11:20

Speaker: Kohji Matsumoto
The Riesz Mean of the Convolution Product of Von Mangoldt Functions and the Related Zeta-function

Time: 11:30-12:00

Speaker: Katsuya Miyake
Twists of Hessian Elliptic Curves and Cubic Fields

Lunch: 12:00 Shandong University Academic Center

Afternoon Session
Chair: Shigeru Kanemitsu

Time: 14:30-15:00

Speaker: Leo Murata
On a Property of the Multiplicative Order of $a \pmod p$

Time: 15:20-15:50

Speaker: Yonggao Chen
On the Prime Power Factorization of $n!$

Time: 16:00-16:20

Speaker: Yoshio Tanigawa
Kronecker's Limit Formula and the Hypergeometric Function

Time: 16:25-16:35

Speaker: Guangshi Lü
Some Results in Classical Analytic Number Theory

Time: 16:40-16:50

Speaker: Huaning Liu
Mean Value of Dirichlet L-functions and Applications to Pseudorandom Binary Sequences in Cryptography

Time: 16:55-17:05

Speaker: Hailong Li
The Structural Elucidation of Eisenstein's Formula

Conference Dinner: 18:30 Shandong University Academic Center

ABSTRACT

Shigeki Akiyama; Niigata University, Japan
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Title: Rational Based Number System and Mahler's Problem

Abstract: We introduce a number system with a fixed rational number base. For instance, it is based by powers of $3/2$ with digits in $\{0, 1, 2\}$ with some restrictions. This is also produced by a boosted adding machine. The system gives a way to expand positive integers. Though the associated language is not even context free, the odometer is given by an automaton.

The system can be "compactified" by extending to right. Each real number is *aperiodic* by this expansion. Up to countable exceptions, the expansion is unique. We characterized this countable exceptions and found an interesting connection with Mahler's problem on the distribution of fractional parts of $(3/2)^n$ in [2].

This talk is based on a joint work [1] with Ch. Frougny (Paris 7) and J. Sakarovitch (CNRS, ENST).

References

[1] S. Akiyama, Ch. Frougny, and J. Sakarovitch, *Powers of rationals modulo 1 and rational base number systems*, submitted.
[2] K. Mahler, *An unsolved problem on the powers of* $3/2$, J. Austral. Math. Soc. **8** (1968), 313–321.

Krishnaswami Alladi; University of Florida, USA
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Title: New Approaches to Jacobi's Triple Product Identity and A Quadruple Product Extension

Abstract: Jacobi's triple product identity for theta functions is one of the most fundamental in the theory of special functions and q-hypergeometric series. We will describe new combinatorial approaches to Jacobi's identity via reformulations of an important partition theorem of Göllnitz. Recently Andrews, Berkovich and I obtained a deep four parameter extension of the theorem of Göllnitz. We will show how specializations of this four parameter theorem can be used to construct a quadruple product extension of the triple product identity.

Tianxin Cai; Zhejiang University, China
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Title: On the Height of Happy Numbers

Abstract: (Joint work with Xia Zhou) In this paper the authors extend a curious harmonic congruence to arbitrary length for any prime $p > 3$ and positive integer $n \le p-2$:

$$\sum_{\substack{l_1+l_2+\cdots+l_n=p\\ l_1,\cdots,l_n>0}} \frac{1}{l_1 l_2 \cdots l_n} \equiv \begin{cases} -(n-1)!\ B_{p-n} (\operatorname{mod} p) & \text{if } 2 \nmid n, \\ -\frac{n}{2(n+1)}\ n!\ B_{p-n-1}\ p\ (\operatorname{mod} p^2) & \text{if } 2|n. \end{cases},$$

We also give a general method to find the least happy number of any given height theoretically, which related to a problem asked by R.Guy.

Yonggao Chen; Nanjing Normal University, China
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Title: (Joint work with Wei Liu) On the Prime Power Factorization of $n!$

Abstract: Let $p_1, p_2, \cdots$ be the sequence of all primes in ascending order. For a positive integer n, let $e_{p_i}(n)$ be the nonnegative integer with $p_i^{e_{p_i}(n)} \mid n$ and $p_i^{e_{p_i}(n)+1} \nmid n$. In 1997, D. Berend proved a conjecture of Erdős and Graham by showing that for every positive integer k there exist infinitely many positive integers n with

$$e_{p_1}(n!) \equiv 0(mod\ 2), e_{p_2}(n!) \equiv 0(mod\ 2), \cdots, e_{p_k}(n!) \equiv 0(mod\ 2).$$

In 2003, Y. G. Chen proved Sander's conjecture: for any given positive integer k and any $\varepsilon_i \in \{0,1\} (i = 1, 2, \cdots, k)$, there exist infinitely many positive integers n such that

$$e_{p_1}(n!) \equiv \varepsilon_1(mod\ 2), e_{p_2}(n!) \equiv \varepsilon_2(mod\ 2), \cdots, e_{p_k}(n!) \equiv \varepsilon_k(mod\ 2).$$

In 2003, F. Luca and P. Stănică posed a conjecture:

Conjecture (F. Luca and P. Stănică). *Let $p_1, \cdots, p_k$ be distinct primes, $m_1, \cdots, m_k$ be arbitrary positive integers ($\geqslant 2$), and $0 \leqslant a_i \leqslant m_i - 1$ for $i = 1, \cdots, k$ be arbitrary residue class modulo m_i. Then*

$$\mid \{0 \leqslant n < N : e_{p_i}(n) \equiv a_i(mod\ m_i), 1 \leqslant i \leqslant k\} \mid \sim \frac{N}{m_1 \cdots m_k} \quad as\ N \to \infty.$$

We will talk about the progress on this topic.

YoungJu Choie; Pohang University of Science and Technology, Korea
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Title: Broué-Enguehard Maps and Atkin-Lehner Involutions

Abstract: (Joint work with P. Solé) Let ℓ be one of the ten integers such that the sum of their divisors divide 24. For each such ℓ, (except 15) we give

a map from an algebra of polynomial invariants of some finite group to the algebra of modular forms invariant under the Atkin-Lehner group of level ℓ. These maps are motivated and inspired by constructions of modular lattices from self-dual codes over rings . This work generalizes Broué-Enguehard work in level one and three obtained from binary and ternary codes.

Vladimir N. Chubarikov; Moscow State University, Russia
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Title: Trigonometric Sums in Number Theory, Analysis and Probability Theory

Abstract: TBA

Kentaro Ihara; Kinki University, Japan
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Title: On the Structure of Algebra of Multiple Zeta Values

Abstract: (Joint work with H. Ochiai) The *multiple zeta value* (MZV) is a number defined by convergent series

$$\zeta(k_1, k_2, \ldots, k_n) = \sum_{m_1 > m_2 > \cdots > m_n > 0} \frac{1}{m_1^{k_1} m_2^{k_2} \cdots m_n^{k_n}} \in \mathbf{R},$$

where $k_1, k_2, \ldots, k_n$ are positive integers with $k_1 > 1$. Here n is the *depth* and $k = k_1 + \cdots + k_n$ the *weight* of the MZV. Let $\mathcal{Z} = \bigoplus_{k \geq 0} \mathcal{Z}_k$ be the graded $\mathbf{Q}$-vector space generated by MZVs, where $\mathcal{Z}_k$ is the component of weight k. It is known $\mathcal{Z}$ has a filtered graded $\mathbf{Q}$-algebra structure: $\mathcal{Z}_k^{(n)} \mathcal{Z}_{k'}^{(n')} \subset \mathcal{Z}_{k+k'}^{(n+n')}$, where $\mathcal{Z}_k^{(n)}$ is the subspace spanned by MZVs of weight k and depth $\leqslant n$. The algebra $\mathcal{Z}$ is deeply connected with the theory of mixed motives and Galois representation on the fundamental group of a curve. The main problem is what the structure of $\mathcal{Z}$ is. Let $D_{k,n}$ be the minimum number of algebra generators of $\mathcal{Z}$ in weight k and depth n.

Theorem For even d, we have $D_{d+3,3} \leqslant \left[\frac{d^2-1}{48}\right]$.

In [1], Kaneko, Zagier and Ihara define a $\mathbf{Q}$-vector space $DSh_n(d)$ (called *double shuffle space*) whose dimension gives an upper bound of the numbers $D_{d+n,n}$ for each n, d. The $DSh_n(d)$ consists of homogeneous polynomials of degree d in n variables with $\mathbf{Q}$-coefficients and which satisfy certain identities. These identities are connected with two kinds of product structures (shuffle products) in $\mathcal{Z}$. For example, $f \in DSh_3(d) \subset \mathbf{Q}[x_1, x_2, x_3]$ satisfies the relations $f(x_1, x_2, x_3) + f(x_2, x_1, x_3) + f(x_2, x_3, x_1) = 0$ and $f^\sharp(x_1, x_2, x_3) + f^\sharp(x_2, x_1, x_3) + f^\sharp(x_2, x_3, x_1) = 0$, where $f^\sharp(a, b, c) = f(a +$

$b+c, b+c, c)$. It is shown in [1] that $DSh_n(d) = \{0\}$ for odd d, consequently $D_{k,n} = 0$ for k, n with distinct parity. For case $n = 2$, it is known that $D_{d+2,2} \leqslant [\frac{d}{6}]$ for even d. For case $n = 3$, Goncharov shows the inequality $D_{d+3,3} \leqslant [\frac{d^2-1}{48}]$ for even d in [2]. Our theorem above gives the same estimate, but by quite different method, using the theory of invariants for reflection groups. As a corollary we have an expression of the space $DSh_3(d)$ in terms of the invariants under the reflection groups.

References

[1] K. Ihara, M. Kaneko, D. Zagier, *Derivations and double shuffle relations for multiple zeta values*, Compositio Math. **142** (2006), 307–338.

[2] A. B. Goncharov, *Multiple polylogarithms, cyclotomy and modular complexes*, Math. Res. Lett. **5** (1998), 497–516.

Chaohua Jia; The Chinese Academy of Sciences, China
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Title: On a Conjecture of Yiming Long

Abstract: TBA

Koichi Kawada; Iwate University, Japan
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Title: On the Sum of Five Cubes of Primes

Abstract: Last year, Mikawa and Peneva proved that if $c > 2/3$ and N is sufficiently large, then all but $o(N^c)$ odd natural numbers in the interval $[N, N + N^c]$ can be written as the sum of five cubes of primes. Working jointly with them, I recently showed the corresponding conclusion with the weaker constraint $c > 7/12$, and I intend to talk on this slight improvement.

Yoshiyuki Kitaoka; Meijo University, Japan
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Title: Distribution of Units of an Algebraic Number Field

Abstract: We study distribution of units modulo a rational prime numbers of an algebraic number field.

Takako Kuzumaki Kobayashi; Gifu University, Japan
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Title: A Transformation Formula for Certain Lambert Series

Abstract: The purpose of the present talk is to deduce the transformation formula for certain Lambert series under the action of the full modular group from the functional equation for the appropriate zeta-functions with sign.character in the spirit of Goldstein-de la Torre.

Winfried Kohnen; Universitaet Heidelberg, Germany
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Title: Sign Changes of Fourier Coefficients and Hecke Eigenvalues of Cusp Forms

Abstract: This is a survey talk in which we will report on recent results regarding changes of signs of Fourier coefficients and Hecke eigenvalues of holomorphic cusp forms in one variable.

Yuk-Kam Lau; The University of Hong Kong, Hong Kong
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Title: The Error Terms in Dirichlet's Divisor Problem, the Circle Problem and the Mean Square Formula of the Riemann Zeta-function.

Abstract: We are concerned with some recent work, contributed by various researchers, on the error terms in the title, which are respectively defined as

$$\begin{aligned}
\Delta(x) &:= \sum_{n\le x} d(n) - x(\log x + 2\gamma - 1),\\
P(x) &:= \sum_{n\le x} r(n) - \pi x,\\
E(t) &:= \int_0^t |\zeta(1/2+iu)|^2\,du - t\biggl(\log\frac{t}{2\pi} + 2\gamma - 1\biggr).
\end{aligned}$$

In addition, we shall discuss on the generalizations of $\Delta(x)$ and $E(t)$.

Hailong Li; Weinan Teacher's College, China
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Title: The Structural Elucidation of Eisenstein's Formula

Abstract: We shall give a complete structural description of the classical Eisenstein formula (and generalizations thereof) that expresses the first periodic Bernoulli polynomial as a finite combination of cotangent values, as a relation between two bases of the vector space of periodic Dirichlet series. We shall also determine the limiting behavior of them, giving rise to Gauss' famous closed formula for the values of the digamma function at rational

points on the one hand and elucidation of Eisenstein-Wang's formulas on the other.

Huaning Liu; Northwest University, China
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Title: Mean Value of Dirichlet L- functions and Applications to Pseudorandom Binary Sequences in Cryptography

Abstract: Let $q \geq 2$, $m \geq 1$ and $n \geq 1$ be positive integers with $m \equiv n \bmod 2$. Set $\epsilon_{m,n} = 1$ if $m \equiv n \equiv 1 \bmod 2$ and $\epsilon_{m,n} = 0$ if $m \equiv n \equiv 0 \bmod 2$. Using generalized Dedekind sums, Bernoulli polynomials and Bernoulli numbers, we prove a formula for the mean value of $L(m,\chi)L(n,\overline{\chi})$ as following:

$$\frac{2}{\phi(q)} \sum_{\substack{\chi \bmod q \\ \chi(-1)=(-1)^m}} L(m,\chi)L(n,\overline{\chi})$$

$$= \frac{(-1)^{\frac{m-n}{2}}(2\pi)^{m+n}}{2m!n!} \left(\sum_{l=0}^{m+n} r_{m,n,l}\phi_l(q)q^{l-m-n} - \frac{\epsilon_{m,n}}{q} B_m B_n \phi_{m+n-1}(q) \right),$$

where

$$r_{m,n,l} = B_{m+n-l} \sum_{\substack{a=0 \\ a+b \geq m+n-l}}^{m} \sum_{b=0}^{n} B_{m-a}B_{n-b} \frac{\binom{m}{a}\binom{n}{b}\binom{a+b+1}{m+n-l}}{a+b+1},$$

B_m is the Bernoulli number, and $\displaystyle\binom{m}{a} = \frac{m!}{a!(m-a)!}$.

Furthermore, for an odd prime p, define

$$e_n = \begin{cases} (-1)^{n+\overline{n}}, & \text{if } n \text{ is a quadratic residue mod } p; \\ (-1)^{n+\overline{n}+1}, & \text{if } n \text{ is a quadratic nonresidue mod } p, \end{cases}$$

where $\overline{n}$ is the multiplicative inverse of n modulo p such that $1 \leq \overline{n} \leq p-1$. We show that the sequence $\{e_n\}$ is a "good" pseudorandom sequence, by using the properties of exponential sums, character sums, Kloosterman sums and mean value theorems of Dirichlet L- functions.

Zhiguo Liu; East China Normal University, China
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Title: A Theta Function Identity to the Quintic Base

Abstract: We prove an identity connecting a theta function and a sum of Eisenstein series by using the complex variable theory of elliptic functions and one theta function identity of S. McCullough and L.-C. Shen, which contains as a special case a famous identity of Ramanujan connected with partitions modulus 5. This identity allows us to develop a theory for Eisenstein series to the quintic base, which is much simpler than that of Chan and Liu [2006, Pacific J. Math 226, 53-64]. By the way, we find a recurrence relation for the Dirichlet series $L(s, \chi_5)$ at even points, which can be used to compute the special values of $L(s, \chi_5)$. An identity for the Bernoulli numbers is also derived. Finally, combining this identity with two identities in Ramanujan's lost notebook, we set up a curious Eisenstein series identity related to the Rogers-Ramanujan continued fraction.

Guangshi Lü; Shandong University, China
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Title: Some Results in Classical Analytic Number Theory

Abstract: In this talk I want to report one new estimate for exponential sums over primes in short intervals and its several applications. These results are as good as what one can obtain from the Generalized Riemann Hypothesis.

Kohji Matsumoto; Nagoya University, Japan
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Title: The Riesz Mean of the Convolution Product of von Mangoldt Functions and the Related Zeta-function

Abstract: Let $\Lambda(n)$ be the von Mangoldt function, that is, $\Lambda(n) = \log p$ if n is a power of a prime p and $\Lambda(n) = 0$ otherwise. The function

$$G_2(n) = \sum_{k+m=n} \Lambda(k)\Lambda(m)$$

is interesting in connection with the famous conjecture of Goldbach, and has been studied by Hardy-Littlewood and Fujii. Fujii proved an asymptotic formula for the summatory function $\sum_{n \le x} G_2(n)$, under the assumption of the Riemann hypothesis (RH).

In this talk, I will report some results, proved in a recent joint work with Shigeki Egami, on the Dirichlet series

$$\Phi_2(s) = \sum_{n=1}^{\infty} \frac{G_2(n)}{n^s}.$$

This series can be regarded as a kind of double zeta-functions, and the Mellin-Barnes integral formula can be applied. We show (under RH) that $\Phi_2(s)$, originally convergent absolutely in $\Re s > 2$, can be continued meromorphically to $\Re s > 1$. Moreover, under certain plausible conjectures, we show that $\Re s = 1$ is the natural boundary of $\Phi_2(s)$.

As an application, we prove an asymptotic formula for Riesz means of $G_2(n)$, whose error estimate is probably best-possible.

Katsuya Miyake; Waseda University, Japan
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Title: Twists of Hessian Elliptic Curves and Cubic Fields

Abstract: In 1840's Hesse investigated plane curves in a series of papers, and found an interesting family of elliptic curves, $H_\mu : U^3 + V^3 + W^3 = 3\mu UVW$, with points of order 3 on the projective plane $\mathbb{P}^2(U : V : W)$. Our concern in this talk is to introduce twists of the curves with μ in the rational number field $\mathbb{Q}$ over quadratic fields, cubic fields and the Galois closures of the latters. We utilize a cubic polynomial, $R(t; X) := X^3 + tX + t, t \in \mathbb{Q} - \{0, -27/4\}$, to parametrize all of quadratic fields and cubic ones; we exclude 0 and $-27/4$ as values of t to save $R(t; X)$ from multiple roots; indeed, its discriminant is $-t^2(4t + 27)$. Let ξ be a root of $R(t; X) = 0$ in the complex number field $\mathbb{C}$ and put $K_t = \mathbb{Q}(\xi)$; if $R(t; X) = 0$ has a root r in $\mathbb{Q}$, then we pick it up as $\xi = r$ and have $K_t = \mathbb{Q}$. Let $\tilde{K}_t$ be the splitting field of the cubic polynomial $R(t; X)$ over $\mathbb{Q}$. If t runs over all such rational values as $K_t = \mathbb{Q}$, then $\tilde{K}_t$ covers all quadratic fields beside $\mathbb{Q}$.

First we define a family $\tilde{H}(\mu, t), \mu, t \in \mathbb{Q}$, of curves of genus 1 which is a twist of H_μ as an algebraic curve over the splitting field $\tilde{K}_t$. Put

$$\Xi = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -t & -t & 0 \end{pmatrix}$$

and $M(x, y, z) := x1_3 + y\Xi + z\Xi^2$ where 1_3 is the unit matrix of size 3. Then we define our $\tilde{H}(\mu, t)$ by

$$\tilde{H}(\mu, t) : \mathrm{Tr}(M(x, y, z)^3) = 3\mu \mathrm{Det}(M(x, y, z))$$

where Tr and Det are the trace and the determinant of matrices, respectively. In general, the curve $\tilde{H}(\mu, t)$, though defined over $\mathbb{Q}$, may not have any rational points over $\mathbb{Q}$. If $K_t = \mathbb{Q}$, however, it has at least one rational point over $\mathbb{Q}$, and hence is an elliptic curve defined over $\mathbb{Q}$; it is a quadratic twist of H_μ if $\tilde{K}_t$ is a quadratic field.

We also show that the curve $H(\mu, t) : 2u^3 + 6d_0uv^2 + w^3 = 3\mu(u^2 - d_0v^2)w,\ d_0 = -(4t + 27)$, is a quadratic twist of H_μ over $\mathbb{Q}(\sqrt{d_0})$ and that it is also isomorphic to $\tilde{H}(\mu, t)$ over K_t.

In the special case of $\mu = 0$ we can show

Proposition 1. Suppose that the cubic polynomial $R(t; X)$ is irreducible over $\mathbb{Q}$ for $t \in \mathbb{Q} - \{0, -27/4\}$. Then the curve $\tilde{H}(0, t)$ has a rational point over $\mathbb{Q}$ if and only if $t = h^3/(2h - 1)^2$ for some $h \in \mathbb{Q} - \{0, 1/2\}$.

We have some one parameter subfamilies of $\tilde{H}(\mu, t)$ which have rational points over $\mathbb{Q}$.

Proposition 2. Suppose that the cubic polynomial $R(t; X)$ is irreducible over $\mathbb{Q}$ for $t \in \mathbb{Q} - \{0, -27/4\}$. Then $\tilde{H}(\mu, t)$ has a rational point over $\mathbb{Q}$ in the following three cases, (1) $t = 3(\mu - 1)$, (2) $t = (1 - \mu)/3$, and (3) $t = 3(1 - \mu)/2$.

In deed, it is clear that, for each $t \in \mathbb{Q} - \{0, -27/4\}$, we have infinitely many such μ as $\tilde{H}(\mu, t)$ has a rational point over $\mathbb{Q}$, and vice versa.

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Title: On a Property of the Multiplicative Order of $a \pmod p$

Abstract: (Joint work with Koji Chinen and and C. Pomerance) For a fixed natural number $a \geq 2$, let $D_a(p)$ denote the residual order of a in $(\mathbf{Z}/p\mathbf{Z})^*$, $\mathbf{P}$ and $\mathbf{N}$ be the set of all prime numbers and the set of all natural numbers, respectively. It is known that the map D_a is (almost) surjective from $\mathbf{P}$ to $\mathbf{N}$, with the property "$|\{D_a^{-1}(n)\}|$ is always finite". In order to study the property of the function D_a more closely, here we consider two types of sets :

$$Q_a(x; k, l) = \{p \leq x;\, p \in \mathbf{P}, D_a(p) \equiv l \pmod k\}, \quad 0 \leq l < k \in \mathbf{N},$$
$$M_2(x) = \{p \leq x;\, p \in \mathbf{P}, D_2(p) \in \mathbf{P}\}.$$

Under the assumption of Generalized Riemann Hypothesis, for any residue class in $\mathbf{N}$, $Q_a(x; k, l)$ has the natural density $\Delta_a(k, l)$, and this value is effectively computable (joint work with K. Chinen). We discuss about some number theoretical properties of $\Delta_a(k, l)$ as a number theoretical function of k and l.

And, as for the second set, we obtain an estimate $|M_2(x)| \ll x(\log x)^{-2}$ which seems to be a best possible estimate.

Kenji Nagasaka; Hosei University, Japan
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Title: Density, Probability and Applications

Abstract: (Joint work with Greco Grecos, Rita Juliano) Densities are, generally speaking, useful tools to solve or to represent mathematical problems, especially to number theoretical problems. Without Schnirelemann densitiy notion, which, at a glance, seems curiously defined, additive number theory, in particular, the theory of base, would not develop as the present situation.

We would like to review several densities, and how they are applied and efficient.

We also report the definition of probabilities, that, indeed, is able to consider a kind of density, and several relating problems, such as, Solobay's measurable cardinals, Banach-Tarski's paradox, occurring events of probability zero, etc. We conclude our presentation by returning Benford's law and Steinhauss' Problem.

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Title: TBA

Abstract: TBA

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Title: Estimates of Exponential Sums over Primes and Applications in Number Theory

Abstract: Exponential sums appear naturally in number theory. Nontrivial estimates of such sums lead to various equi-distribution theorems, as well as solutions to problems in additive number theory. For example, the solution of the famous Waring-Goldbach problem depends on estimates of exponential sums over primes. A celebrated example is Vinogradov's proof of the ternary Goldbach conjecture.

In this talk, by using different methods, we will give such estimates for exponential sums over κ-th powers of primes, where κ is a positive integer or $\kappa \in (0,1)$. Our theorem covers that of Vinogradov in the case $0 < \kappa \leq 1$, and are new in all the other (nonlinear) cases. Applications of these estimates in number theory will be illustrated.

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Title: The Number of Solutions in a Box of a Linear Homogeneous Congruence

Abstract: **Theorem.** Let a_i, b_i $(i = 1, \dots, k)$ be integers, $b_i > 0$, $n = \prod_{j=1}^{l} q_j^{\alpha_j}$, q_j distinct primes, $\alpha_j > 0$. If

$$\sum_{j=1}^{l} \frac{1}{q_j} \leq 1 + \frac{\min\{l, 2l-5\}}{n},$$

then the number of solutions of the congruence $a_1x_1+\ldots+a_kx_k \equiv 0 \pmod n$ such that $0 \le x_i \le b_i$ is at least $2^{1-n}\prod_{i=1}^{k}(b_i+1)$.

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Title: Exponential and Character Sums with Combinatorial Sequences

Abstract: We give a survey of recent results, obtained in collaboration with Moubariz Garaev and Florian Luca, on bounds of exponential and character sums with various sequences of combinatorial interests such as

- middle binomial coefficients $\mathcal{B} = \left(\binom{2n}{n}\right)_{n=1}^{\infty}$,
- Catalan numbers $\mathcal{C} = \left(\frac{1}{n+1}\binom{2n}{n}\right)_{n=1}^{\infty}$,
- factorials $\mathcal{F} = (n!)_{n=1}^{\infty}$.

More precisely, given a sequence $\mathcal{U} = (u_n)$ and a prime p, we define exponential sums

$$\sum_{n=1}^{N}\chi(u_n), \qquad \sum_{m=1}^{M}\sum_{n=1}^{N}\exp(2\pi i a u_m u_n/p), \qquad \sum_{n=1}^{N}\chi(u_n)$$

with an integer a relatively prime to p and a nontrivial multiplicative character χ modulo p.

We present several recent estimates obtained for these sums with the above sequences. For example, we prove that

$$\sum_{n=1}^{N}\chi(u_n) = O(N^{3/4}p^{1/8}(\log p)^{1/4}), \qquad 1 \le N < p,$$

where $\mathcal{U} = (u_n)$ is one of the sequences $\mathcal{B}$, $\mathcal{C}$ or $\mathcal{F}$. which is nontrivial for $N \ge p^{1/2+\varepsilon}$. We also discuss some results about the number of solutions of various congruences with these sequences $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{F}$ which could be of independent interest.

Finally we indicate some applications, for example, to the Waring problem with factorials and outline some open problems.

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Title: On the Exponential Equations $a^x - b^y = c\,(1 \le c \le 300)$

Abstract: M. A. Bennett obtained the following theorem in 2001. If a, b, c are positive integers with with a, $b \ge 2$ and $1 \le c \le 100$, then the equation

$a^x - b^y = c$ has at most one solution in positive integers x and y except the ten exceptional cases. Now we will study the same problem in the case of $1 \le c \le 300$. We follow Bennett's method.

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Title: Curious Identities and Congruences Involving Bernoulli Polynomials

Abstract: In this talk we first tell the story how the developments of some curious identities concerning Bernoulli polynomials finally led to the following unified symmetric relation (due to Z. W. Sun and H. Pan): If n is a positive integer, $r+s+t=n$ and $x+y+z=1$, then we have

$$r\begin{pmatrix} s & t \\ x & y \end{pmatrix}_n + s\begin{pmatrix} t & r \\ y & z \end{pmatrix}_n + t\begin{pmatrix} r & s \\ z & x \end{pmatrix}_n x = 0$$

where

$$\begin{pmatrix} s & t \\ x & y \end{pmatrix}_n := \sum_{k=0}^{n}(-1)^k \binom{s}{k}\binom{t}{n-k} B_{n-k}(x)B_k(y).$$

Let p be a prime, and let $n > 0$ and r are integers. We will also talk about some congruences involving the Felck quotient

$$F_p(n,r) := (-p)^{-\lfloor (n-1)/(p-1)\rfloor} \sum_{k\equiv r \ (\bmod p)} \binom{n}{k}(-1)^k \in \mathbb{Z}$$

and Bernoulli polynomials, obtained by p-adic methods. Finally we mention some related applications to Stirling numbers of the second kind and homotopy exponents of special unitary groups.

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Title: Kronecker's Limit Formula and the Hypergeometric Function

Abstract: We show a new proof of Kronecker's (first) limit formula using the hypergeometric function. First, we divide the Epstein zeta function into two parts. Each part can be written by the integral of the hypergeometric function,but the argument of one part lies in a branch different from the standard one. To treat this part, we make use of the connection formula of the hypergeometric function.

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Title: On Gauss' Formula for ψ and Finite Expressions for the L-series at 1

Abstract: We show that Gauss' closed formula for the values of the digamma function at rational points is equaivalent to the finite expression for the $L(1,\chi)$, which appears in the class number formula for quadratic fields. We also give several equivalent expressions for $N(q)$ (Theorem 2) introduced by Lehmer.

Haruo Tsukada; Kinki University, Japan
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Title: A General Modular Relation Associated with the Riemann Zeta-function

Abstract: In this talk, we would like to present a modular relation of an extremely general form, associated with the Riemann zeta-function. This formula includes the Bochner's formula, the Riesz sum formula, a K-Bessel expansion, the incomplete Gamma expansion, and a J-Bessel expansion as special cases, and enables us to treat wide variaties of similar formulas in a systematic way.

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Title: Some Thue Equations and Continued Fractions

Abstract: We consider the parameterized families of cubic and quartic Thue equations

$$bx^3 - ax^2y - (a+3b)xy^2 - by^3 = k,$$
$$bx^4 - ax^3y - 6bx^2y^2 + axy^3 + by^4 = k.$$

We solve these equations when k is small and a is sufficiently large compared with b. We use Pad'e approximation method to bound from above the size of the solutions (x, y).

Daqing Wan; University of California, USA
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Title: L-Functions of Infinite Symmetric Powers

Abstract: This is an expository lecture on L-functions of function fields. For a finite dimensional l-adic representation, the L-function is theoretically well understood by celebrated works of Grothendieck and Deligne. Our goal here is to understand the p-adic variation and its limit (if exists) of the sequence of L–function of the k-th symmetric power of a geometric

representation as k goes to infinity but converges p-adically to a p-adic integer. In the universal elliptic curve example, the p-adic limit is essentially the theory of overconvergent p-adic modular forms. In general, it plays an essential role in the proof of Dwork's conjecture. It should relate closely to the conjectual theory of p-adic automorphic forms which is yet to be developed.

Trevor D. Wooley; University of Michigan, USA
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Title: Waring's Problem in Function Fields

Abstract: (Joint work with Yu-Ru Liu (Waterloo)) Let F be a finite field of characteristic p, let K denote the polynomial ring $F[t]$, and write $J[t]$ for the additive closure of the set of kth powers of polynomials in K. Define $G_q(k)$ to be the least integer s satisfying the property that every polynomial in $J[t]$ of sufficiently large degree admits a strict representation as a sum of s kth powers. We employ a version of the Hardy-Littlewood method involving the use of smooth polynomials inorder to establish a bound of the shape $G_q(k) \leq Ck \log k + O(k \log \log k))$. Here, the coefficient C is equal to 1 when $k < p$, and C is given explicitly in terms of k and p when $k > p$, but in any case satisfies $C \leq 4/3$. There are associated conclusions for the solubility of diagonal equations over K, and for exceptional set estimates in Waring's problem.

Honggang Xia; Shandong University, China
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Title: On Zeros of Cubic L-functions.

Abstract: We study the distribution of zeros of cubic L-functions and obtain a zero density theorem by large sieve method. By Patterson's work on cubic Gauss sums, we get an estimation on the moments of corresponding L-functions and result of class numbers of a certain type of number fields follows as an application.

Yangbo Ye; University of Iowa, USA
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Title: A New Bound for Rankin-Selberg L–functions

Abstract: This talk will report a subconvexity bound for $L(1/2, f \times g)$ in the weight eigenvalue aspect of f, recently proved by Yuk-Kam Lau, Jianya Liu, and the speaker. The new bound is $O(k^{2/3+\epsilon})$, while the trivial, convexity bound in this case is $O(k)$. This result is believed to have

reached the second barrior, after the convexity bound, toward the Lindelof Hypothesis. Its applications include an estimate for the fourth power moment of $L(1/2 + it, g)$ over a short interval of t. The techniques used in the proof include the Kuznetsov trace formula, spectral large sieving, Good's estimation, spectral analysis and meromorphic continuation of a shifted convolution sum, and weighted stationary phase.

Wenguang Zhai; Shandong Normal University, China
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Title: On the Fourth Power Moments of $\Delta(x)$ and $E(t)$

Abstract: Let $\Delta(x)$ and $E(t)$ denote the error term in the Dirichlet divisor problem and the error term in the mean square of $\zeta(1/2 + it)$, respctively. It is an important problem in the analytic number theory to study the power moments of these two error terms. In this talk, we will give two new results about the fourth power moments of these two functions, which further improve previous results.

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Title: Zero Density Estimates for Automorphic L-functions

Abstract: In this talk, I will give zero density estimates of the large sieve type for the automorphic L-functions $L(s, f \bigotimes \chi)$, where f is a holomorphic cusp form for the group $SL_2(Z)$ and $\chi(\mathrm{mod} q)$ is a primitive Dirichlet character.

Shou-Wu Zhang; Columbia University, USA
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Title: Periods Integrals and Special Values of L-series

Abstract: I will survey some basic results about periods integrals of automorphic forms and their relations with special values of L-series.

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Title: The Mean Value of Dedekind Sum and Cochrane Sum in Short Intervals

Abstract: For a positive integer q and an arbitrary integer h, the classical Dedekind sum $S(h,q)$ and Cochrane sum $C(h,q)$ are defined by

$$S(h,q)=\sum_{a=1}^{q}\left(\left(\frac{a}{q}\right)\right)\left(\left(\frac{ah}{q}\right)\right),$$

where

$$((x))=x-[x]-\frac{1}{2},\text{ if } x \text{ is not an integer};\quad ((x))=0 \text{ if } x \text{ is an integer},$$

$$C(h,q)=\sum_{a=1}^{q}{}'\left(\left(\frac{\overline{a}}{q}\right)\right)\left(\left(\frac{ah}{q}\right)\right),$$

where $\overline{a}$ is defined by the equation $a\overline{a}\equiv 1 \mod q$ and $\sum_{a=1}^{q}{}'$ denotes the summation over all $1\le a\le q$ such that $(a,q)=1$. What is the mean value distribution properties of the Dedekind sum or Cochrane sum in a short interval $[1,\lambda p]$, where $\lambda\in(0,1)$. In fact, it is very difficult to get a asymptotic formula for the mean value in the case of one variety as

$$\sum_{a<\frac{p}{4}}S(a,p)\qquad\text{and}\qquad\sum_{a<\frac{p}{4}}C(a,p).$$

The main purpose of this paper is to study a special kind of mean value properties of the Dedekind sum and Cochrane sum by using the mean value theorems of the Dirichlet L-functions, and prove the following results:

$$\sum_{a<\frac{p}{4}}\sum_{b<\frac{p}{4}}S(a\bar{b},p)=\frac{3}{64}p^2+O\left(p^{1+\epsilon}\right)$$

and

$$\sum_{a<\frac{p}{4}}\sum_{b<\frac{p}{4}}C(ab,p)=\frac{45}{1024}p^2+O\left(p^{1+\epsilon}\right).$$